New preservation properties for stochastic orderings and aging classes under the formation of order statistics and systems

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The study of preservation properties for stochastic orderings and aging classes under the formation of order statistics is a relevant topic in the literature. Thus, it is well known that some stochastic orderings and aging classes are not preserved under the formation of order statistics. Hence some conditions are needed in order to guarantee these preservation properties which are important to obtain related properties (bounds, inference, ...). The new properties are based in the representation of the distributions of order statistics as distorted distributions (identically distributed case) or generalized distorted distributions (general case). A distorted distribution is a distribution function G which can be written as $G = q(F)$, where F is a baseline distribution function and q is an increasing continuous function in the interval $[0, 1]$ such that $q(0) = 0$ and $q(1) = 1$. The function q is called the *distortion function* and it might contain some parameters. Distorted distributions were firstly used in the context of the rank-dependent expected utility model but have applications in different areas. For example, the popular Proportional Hazard Rate (PHR) model used in the Analysis of Survival Data can be represented as a distorted distribution. The generalized distorted distributions are defined in a similar way through the expression $G = Q(F_1, \ldots, F_n)$, where $F_1 \ldots, F_n$ are baseline distribution functions and Q is an increasing continuous function from $[0, 1]^n$ to $[0, 1]$ such that $Q(0, \ldots, 0) = 0$ and $Q(1, \ldots, 1) = 1$. The distorted distributions can also be used to represent the distributions of coherent systems in the context of the Reliability Theory. Hence the preservation results can also be applied to these systems both in the case of independent or dependent components. In this context, the order statistics represents the lifetimes of k-out-of-n systems (i.e. systems which work if at least k of their n components work).

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