CONFERENCE PROGRAM AND ABSTRACT BOOK

11th INTERNATIONAL CONFERENCE
ON ORDERED STATISTICAL DATA

2-6.06.2014
BĘDLEWO
POLAND

Conference venue

Będlewo Palace
The Mathematical Research and Conference Center
Institute of Mathematics of the Polish Academy of Sciences
Będlewo, ul. Parkowa 1
62-060 Stęszew
Poland

Edited by

Patryk Miziuła

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Contents

Introduction

Continuing the series of Conferences in Mysore, India (2000), Warsaw, Poland (2002–04), Izmir, Turkey (2005), Mashad, Iran (2006), Amman, Jordan (2007), Aachen, Germany (2008), Zagazig, Egypt (2010), Murcia, Spain (2012), the Institute of Mathematics of the Polish Academy of Sciences and Nicolaus Copernicus University will host the 11th International Conference on Ordered Statistical Data OSD 2014. The meeting will be held at the Mathematical Research and Conference Center in Będlewo, Poland, June 2nd–6th, 2014.

The conference will bring forth recent advances and trends in the mathematical theory of ordered statistical data, in order to facilitate the exchange of research ideas, promote collaboration among researchers from all over the world, and contribute to the further development of the field.

The meeting will be dedicated to all aspects of ordered statistical data, including:

- Approximations
- Bounds
- Characterizations
- Recurrence Relations
- Distribution Theory and Probability Models
- Stochastic Orders
- Reliability Theory and Survival Analysis
- Censoring
- Concomitants
- Statistical Interference
- Applications of Ordered Data
- Information and Entropies
- Nonparametric Methods
- Ranked Set Sampling
- Asymptotic Theory

The 11th International Conference on Ordered Statistical Data OSD 2014 is co-organized and co-financed by the Warsaw Center of Mathematics and Computer Science, Stefan Banach International Mathematical Center and Nicolaus Copernicus University in Toruń:

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Plenary talks

Jafar Ahmadi (Ferdowsi University of Mashhad, Iran)
Narayanaswamy Balakrishnan (McMaster University, Canada)
Udo Kamps (RWTH Aachen University, Germany)
Jorge Navarro (University of Murcia, Spain)
Moshe Shaked (The University of Arizona, USA)

Bartoszewicz Memorial Session

On Tuesday morning, June 3, two sessions devoted to the memory of Jarosław Bartoszewicz are planned.

Professor Bartoszewicz who passed away on February 24, 2013, was a leading specialist in the theory of stochastic orderings and a member of our Scientific Committee.

The first session consists of two talks. The first one is delivered by Dr Piotr Nowak, a Ph.D. student of Bartoszewicz, who will present the most significant research achievements of his late supervisor.

The second speaker of the session is Professor Moshe Shaked (The University of Arizona, USA) who will read a one hour plenary lecture on stochastic orders.

The second session on stochastic orders, organized and chaired by Professor Shaked, consists of 4 invited talks. The speakers of the session are:

Professor Abdulhamid Alzaid (King Saud University, Kingdom of Saudi Arabia),
Professor Felíx Belzunce (Universidad de Murcia, Spain),
Professor Alfonso Suárez-Llorens (University of Cádiz, Spain),
Professor Peng Zhao (Jiangsu Normal University, China).

Professor Jarosław Bartoszewicz
(1945-2013)

Jarosław Bartoszewicz was born on July 22, 1945 in Różanka near Nowogródek (now in Belarus). He received his school education in Legnica, a town in Lower Silesia. From 1964 to 1969 he studied mathematics at the Wrocław University and earned his M.Sc. diploma. After graduating, he started working at the Wrocław University, and spent the rest of his career there. In June 1973, he defended his doctoral dissertation *Estimation of reliability in the exponential model*, written under supervision of Bolesław Kopciński. In 1987, he obtained his habilitation degree based on the habilitation thesis *Robust estimation of the scale parameter*. In September 2009, he was nominated by President of Poland for the title of full professor in mathematical sciences. He passed away on February 24, 2013, in Wrocław.

The research career of Jarosław Bartoszewicz was mostly influenced by Bolesław Kopociński and Witold Klonecki, as well as the seminars on applications of mathematics organized by Józef Łukaszewicz. Professor Bartoszewicz worked mainly in the areas of mathematical statistics and reliability theory. He wrote 39 research papers devoted to three groups of problems: unbiased estimation of the exponential reliability, robust scale estimation in parametric models under violations generated by stochastic orders, and characterizations, properties, and applications of stochastic orderings and families of probability distributions generated by stochastic orders.

The most important results obtained by Professor Bartoszewicz concern relationships between classes of lifetime distributions and stochastic orders, in particular characterizations of the star and dispersive orderings via the Laplace transform. Other interesting results are connected with the weighted distributions and related problems of preservation of classes of probability distributions and stochastic orders under weighting. Yet another remarkable contribution is the construction of goodness-of fit tests under the alternatives ordered in the dispersive order. In the monograph *Stochastic orders* (Springer Series in Statistics, 2007) by M. Shaked and J.G. Shanthikumar, 16 papers of Bartoszewicz were cited, and some his theorems were presented with the original proofs.

Jarosław Bartoszewicz organized several Polish national statistical conferences. He was invited for presenting lectures at international conferences in France, India and Turkey. He was the chief co-editor of *Applicationes Mathematicae*, and a member of the Mathematical Statistics Section of the Committee of Mathematics of Polish Academy of Sciences. For his research achievements, he was awarded a prize of Minister of Science, Higher Education and Technology in 1974, and the prestigious Hugo Steinhaus Prize of the Polish Mathematical Society in 2000.

For many years, Professor Bartoszewicz delivered lectures on mathematical statistics and their applications for students. He wrote *Lectures on Mathematical Statistics* (PWN, 1989, 1996), the most competent textbook on mathematical statistics published by Polish author. He supervised more than eighty master theses and promoted five Ph.D. students in mathematics.

Program

Sunday, June 1

17:00–22:00	Registration
18:00–	Dinner

Monday, June 2

8:00– 9:00	Breakfast
8:15– 9:15	Registration
9:15– 9:30	Opening Ceremony
9:30–10:30	Plenary lecture chairman: Haikady N. Nagaraja
	Having fun with Laplace **Narayanaswamy Balakrishnan**
10:35–11:15	*Reliability models, estimation* chairman: Katherine Davies
10:35	*Statistical inference for component distribution from system lifetime data* **Hon Keung Tony Ng**
10:55	*Meta-analysis based on minimal repair times of independent systems* **Morteza Amini**
11:15–11:40	Coffee break
11:40–13:00	*Stochastic orders, reliability* chairman: Morteza Amini
11:40	*Stochastic comparisons of some conditional distributions under dependence* **Miguel A. Sordo**
12:00	*Characterizations of Weibull distributions* *through selected functions of reliability theory* **Magdalena Szymkowiak**
12:20	*Properties of aging intensity and reversed aging intensity functions* **Maria Iwińska**
12:40	*On parallel systems with heterogeneous Weibull components* **Nuria Torrado**

13:00–15:00 Lunch

15:00–16:20 *Records*
chairman: Nickos Papadatos

15:00 ($\delta \geq 0$)-*records*
Fernando López-Blázquez

15:20 *Pólya-Aeppli shocks*
Leda Minkova

15:40 *On concomitants of record values from generalized Farlie-Gumbel--Morgenstern distribution*
Nahed A. Mokhlis

16:00 *Two new maximum likelihood methods to estimate the sample size from the record values*
Inmaculada Barranco-Chamorro

15:00–16:20 *Ranked set sampling*
chairman: Birdal Şenoğlu

15:00 *Estimation of the reliability in the stress-strength models under the modifications of the ranked set sampling: Weibull distribution*
Fatma Gül Akgül

15:20 *Estimation of the inverse Weibull distribution based on progressively censored data: comparative study*
Amal Helu

15:40 *Application of ranked set sampling on correlation coefficient (R) test for normality*
Ayşegül Erem

16:00 *Estimation of harmonic and geometric mean in ranked set sampling*
Sukma Adi Perdana

16:20–16:50 Coffee break

16:50–17:50 *Discontinuous models*
chairman: Eugenia Stoimenova

16:50 *Discrete q-uniform distribution*
Charalambos A. Charalambides

17:10 *Class of estimator for finite population variance under two-phase sampling*
Mohd Saleh Ahmed

17:30 *Association of zero-heavy continuous variables*
Magdalena Niewiadomska-Bugaj

19:00–22:00 Welcome Reception

Tuesday, June 3

8:00– 9:00 Breakfast

9:00–10:30 Bartoszewicz Memorial Session
chairman: Tomasz Rychlik

9:00 *Professor Bartoszewicz's contributions to mathematical statistics and its applications*
Piotr Bolesław Nowak

9:30 *The dispersive stochastic orders: univariate and multivariate*
Moshe Shaked

10:30–11:00 Coffee break

11:00–12:30 Bartoszewicz Memorial Session continued
chairman: Moshe Shaked

11:00 *On ordering the convolution of the difference of Bernoulli random variables*
Abdulhamid Alzaid

11:30 *Some results for the comparison of generalized order statistics in the total time on test and excess wealth orders*
Félix Belzunce

12:00 *A new variability order based on tail-heaviness*
Alfonso Suárez-Llorens

12:30–15:00 Lunch

15:00–16:20	*Stochastic orders* chairman: Miguel A. Sordo	*Estimation methods* chairman: Magdalena Niewiadomska-Bugaj
15:00	*Comparing relative skewness of random vectors* **Julio Mulero**	*Bootstrap method for central and intermediate order statistics under power normalization* **Osama Mohareb Khaled**
15:20	*A class of measures of shape: properties and orderings* **Antonio Arriaza-Gomez**	*Sequential estimation of a common location parameter of two populations* **Agnieszka Stępień-Baran**
15:40	*Preservation of the GTTT transform order under some reliability operations* **Maria Kamińska-Zabierowska**	*A comparison of different methods for estimating the missing value in ANOVA* **Demet Aydin**
16:00	*Sufficient conditions for the total time on test transform order* **Carolina Martínez-Riquelme**	*Inference on $P(Y < X)$ based on record values for generalized Pareto distribution* **Manoj Chacko**

16:20–16:50 Coffee break

16:50–17:50 *Bounds*
chairman: Charalambos A. Charalambides

16:50 *Maximizing the expected range from dependent samples under mean-variance information*
Nickos Papadatos

17:10 *Inequalities for variances of order statistics coming from urn models*
Krzysztof Jasiński

17:30 *Bounds on dispersion measures in Bayesian mixture models*
Patryk Miziuła

19:00–22:00 Barbecue

Wednesday, June 4

8:00– 9:00 Breakfast

9:00–10:00 Plenary lecture
chairman: Tony Ng

New preservation properties for stochastic orderings and aging classes under the formation of order statistics and systems
Jorge Navarro

10:10–11:10 *Censoring schemes, estimation*
chairman: Erhard Cramer

10:10 *Inference under a type-I censoring scheme with self-determined threshold*
George Iliopoulos

10:30 *Type-I censored generalized order statistics*
Marco Burkschat

10:50 *Scale parameter estimation under an order statistics prior*
Maria Kateri

11:10–11:40 Coffee break

11:40–13:00 *Interval estimation and prediction for OSD*
chairman: Marco Burkschat

11:40 *Nonparametric statistical intervals based on ordered data*
Erhard Cramer

12:00 *Prediction for future exponential lifetime based on random number of generalized order statistics under a general set-up*
El-Sayed Mahsoub Nigm

12:20 *Minimum volume confidence regions in models of sequential order statistics with conditional proportional hazard rates*
Jens Lennartz

12:40 *Two-sample Bayesian prediction intervals for progressively type-II censored competing risks data from the half-logistic distribution*
Alaa H. Abdel-Hamid

13:00–13:40 Lunch

14:00–19:00 Excursion

19:00–20:00 Dinner

Thursday, June 5

8:00– 9:00 Breakfast

9:00–10:00 Plenary lecture
chairman: Haroon M. Barakat

Distances between models of generalized order statistics
Udo Kamps

10:10–11:10 *Testing procedures*
chairman: Leda Minkova

10:10 *Homogeneity-testing in multiparameter exponential families*
Alexander Katzur

10:30 *Exceedance-type tests*
Eugenia Stoimenova

10:50 *A test of multidimensional stochastic dominance of I order using multidimensional quantile functions*
Kamil Dyba

11:10–11:40 Coffee break

11:40–13:00 *Near-order statistics observations, characterizations*
chairman: George Iliopoulos

11:40 *Spacings around an order statistic*
Haikady N. Nagaraja

12:00 *Characterizations based on moments of the number of observations near-order statistics*
Massoumeh Fashandi

12:20 *Linearity of regression for overlapping order statistics*
Jacek Wesołowski

12:40 *Characterization of exponential distribution through record values and associated beta distribution*
George Yanev

13:00–15:00 Lunch

15:00–16:20	*Distribution theory* chairman: Nahed A. Mokhlis	*Estimation methods* chairman: El-Sayed M. Nigm
15:00	*A note on the cumulative residual entropy* **Younes Zohrevand**	*Fuzzy-weighted ranked set sampling: a new way to construct ranked set sampling* **Bekir Cetintav**
15:20	*Distribution of the rank of order statistics from bivariate sample* **Gülder Kemalbay**	*Best constant-stress accelerated life-test plans for one-shot devices under budget and time constraints* **Man Ho Ling**
15:40	*Stochastic processes with good memory* **Jerzy Filus**	*Bayes estimation of the Pareto parameters under progressive censoring data for constant-partially accelerated life tests using MCMC* **Tahani A. Abushal**
16:00	*On a class of half-logistic generated distributions* **Mohamed Hussein**	*Stochastic monotonicity of estimators* **Piotr Bolesław Nowak**

16:20–16:50 Coffee break

16:50–17:50 *Asymptotic theory*
chairman: Jacek Wesołowski

16:50 *On univariate extreme value statistics and the estimation of the conditional tail expectation*
Rassoul Abdelaziz

17:10 *Asymptotic distributions of order statistics and record values arising from the gamma and Kumaraswamy-generated-distributions families*
Haroon M. Barakat

17:30 *Limit processes for sequences of partial sums of residuals of regressions against order statistics with Markov-modulated noise*
Artem Kovalevskiy

19:00–22:00 Gala Dinner

Friday, June 6

8:00– 9:00 Breakfast

9:00–10:00 Plenary lecture
chairman: Maria Kateri

Stochastic comparison of repairable systems
Jafar Ahmadi

10:10–10:30 *Records estimation*
chairman: Maria Kateri

10:10 *Exact prediction intervals for future current records and record range from any continuous distribution*
Haroon M. Barakat

10:30–10:50 Coffee break

10:50–11:50 *Bounds for OSD*
chairman: Fernando López-Blázquez

10:50 *Optimal bounds on the bias of quasimidranges*
Mariusz Bieniek

11:10 *Bounds on expectations of L-statistics from maximally and minimally stable samples*
Andrzej Okolewski

11:30 *Optimal bounds on expectations of order statistics and spacings based on the ID distributions*
Agnieszka Goroncy

11:50–12:00 Closing Ceremony

12:30–14:30 Lunch

Abstracts

Plenary talks

Having fun with Laplace

Narayanaswamy Balakrishnan (bala@mcmaster.ca, McMaster University)

The Laplace distribution has a long history, and the likelihood estimation of its parameters has always been an interesting problem. In this talk, I will first present some historical details about the distribution, the work on order statistics from the distribution, and some results concerning the maximum likelihood estimation of the location and scale parameters. I will then describe some interesting recent developments on estimation methods based on censored samples, and present exact inferential results as well as computational results, along with some illustrative examples. Finally, if time permits, I will point out some problems for further research in this direction.

The dispersive stochastic orders: univariate and multivariate

Moshe Shaked (shaked@math.arizona.edu, The University of Arizona)

This talk will be given in the Bartoszewicz Memorial Session. Jarosław Bartoszewicz did a lot of work on stochastic orders such as the convex transform order, the star order, the superadditive order, as well as the Laplace transform order, the Laplace transform ratio order, the reversed Laplace transform ratio order, and other orders.

One order in which Bartoszewicz's contributions are quite remarkable is the dispersive stochastic order. In this talk I will first review the univariate dispersive order. I will then continue in describing some multivariate extensions of the univariate dispersive order. References will be made to some works of Jarosław Bartoszewicz from 1985 through 2001, all involving the dispersion orders, with applications to the orderings of order statistics.

References

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J. Bartoszewicz (1986). Dispersive ordering and the total time on test transformation, *Statistics and Probability Letters* **4**, 285–288.

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J. Bartoszewicz (1996). Tail orderings and the total time on test transform, *Applicationes Mathematicae* **24**, 77–86.

J. Bartoszewicz (2001). Stochastic comparisons of random minima and maxima from life distributions, *Statistics and Probability Letters* **55**, 107–112.

New preservation properties for stochastic orderings and aging classes under the formation of order statistics and systems

Jorge Navarro (jorgenav@um.es, University of Murcia)

The study of preservation properties for stochastic orderings and aging classes under the formation of order statistics is a relevant topic in the literature. Thus, it is well known that some stochastic orderings and aging classes are not preserved under the formation of order statistics. Hence some conditions are needed in order to guarantee these preservation properties which are important to obtain related properties (bounds, inference, ...). The new properties are based in the representation of the distributions of order statistics as distorted distributions (identically distributed case) or generalized distorted distributions (general case). A distorted distribution is a distribution function G which can be written as $G = q(F)$, where F is a baseline distribution function and q is an increasing continuous function in the interval $[0,1]$ such that $q(0) = 0$ and $q(1) = 1$. The function q is called the *distortion function* and it might contain some parameters. Distorted distributions were firstly used in the context of the rank-dependent expected utility model but have applications in different areas. For example, the popular Proportional Hazard Rate (PHR) model used in the Analysis of Survival Data can be represented as a distorted distribution. The generalized distorted distributions are defined in a similar way through the expression $G = Q(F_1,\dots,F_n)$, where $F_1 \dots, F_n$ are baseline distribution functions and Q is an increasing continuous function from $[0,1]^n$ to $[0,1]$ such that $Q(0,\dots,0) = 0$ and $Q(1,\dots,1) = 1$. The distorted distributions can also be used to represent the distributions of coherent systems in the context of the Reliability Theory. Hence the preservation results can also be applied to these systems both in the case of independent or dependent components. In this context, the order statistics represents the lifetimes of k-out-of-n systems (i.e. systems which work if at least k of their n components work).

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J. Navarro, Y. del Aguila, M. A. Sordo, A. Suárez-Llorens (2013). Stochastic ordering properties for systems with dependent identically distributed components, *Applied Stochastic Models in Business and Industry* **29**, 264–278.

J. Navarro, Y. del Aguila, M. A. Sordo, A. Suárez-Llorens (2014). Preservation of stochastic orders under the formation of generalized distorted distributions. Applications to coherent systems and other concepts. Submitted.

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J. Navarro, T. Rychlik (2010). Comparisons and bounds for expected lifetimes of reliability systems, *European Journal of Operational Research* **207**, 309–317.

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Distances between models of generalized order statistics

Udo Kamps (udo.kamps@rwth-aachen.de, RWTH Aachen University)

The Hellinger metric and several divergence measures for multivariate density functions are applied to measure distances between different models of generalized order statistics, such as common order statistics, sequential order statistics, progressively type-II censored order statistics, record values, k-th record values, and Pfeifer record values. Explicit expressions of divergences and distances are shown along with some properties and structural findings within the family of generalized order statistics. Moreover, the results are statistically utilized to find a closest common order statistics model to some given model of sequential order statistics, to construct multivariate confidence regions for the parameter vector of sequential order statistics as well as to test the null hypothesis of common order statistics against a sequential order statistics alternative.

Stochastic comparison of repairable systems

Jafar Ahmadi (ahmadi-j@um.ac.ir, Ferdowsi University of Mashhad)
Majid Chahkandi (ma.chahkandi@yahoo.com, University of Birjand)

The aim of this paper is to study the repairable systems which begins to operate at time 0. If the system fails, then it undergoes minimal repair and begins to operate again. Here minimal repair means that the repair done on a system leaves the system in exactly the same condition as it was just before failure, (see for example Barlow and Proschan, 1966). It is assumed that the number of repair is a random variable, the proposed plan was introduced by Chahkandi et al. (2014). The idea is extended for comparing the performance of two repairable systems. Comparison results for the inactivity and residual lifetimes of two repairable systems are obtained.

References

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Bartoszewicz Memorial Session talks

Professor Bartoszewicz's contributions to mathematical statistics and its applications

Piotr Bolesław Nowak (pnowak@math.uni.opole.pl, Opole University)

In this talk we recall the most important papers and results of Professor Bartoszewicz in the fields of statistical inference, mathematical theory of reliability and stochastic orders.

On ordering the convolution of the difference of Bernoulli random variables

Abdulhamid Alzaid (alzaid@ksu.edu.sa, King Saud University)
Maha A. Omair (maomair@ksu.edu.sa, King Saud University)
Om Alsad Aodah (Prince Salman University)

Recently there is a growing interest on models based on discrete distributions defined on the set of integers Z . In this paper, we consider the ordering of probability distributions generated from the sum of the difference of Bernoulli random variables.

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Inusah, S., Kozubowski, T.J. (2006). A discrete analogue of the Laplace distribution. Journal of Statistical Planning and Inference, 136, 1090-1102.

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Some results for the comparison of generalized order statistics in the total time on test and excess wealth orders

Félix Belzunce (belzunce@um.es, University of Murcia)
Carolina Martínez-Riquelme (carolina.martinez7@um.es, University of Murcia)

This talk is devoted to the study of the comparison of generalized order statistics in terms of the total time on test transform and excess wealth orders. We provide some extensions of previous results in the literature for usual order statistics and generalized order statistics. These results involve results for the minimum of a random vector of generalized order statistics.

Linear combinations of random variables and their applications

Peng Zhao (zhaop07@gmail.com, Jiangsu Normal University)

Linear combinations of random variables have attracted considerable attention due to their typical applications in nonparametric goodness-of-fit tests, economics, game theory, engineering, insurance, information science, reliability, etc. Let $X_1, \ldots, X_n$ be independent random variables on $\mathbb{R}_+$. We are interested in the linear combination, $\sum_{i=1}^{n} a_i X_i$, $a_i \in \mathbb{R}_+$. Especially, the cases when X_i is exponential or gamma distribution are extremely important since they have many interesting applications in various areas. For example, in reliability theory, it naturally arises in the study of redundant standby systems; in queuing theory, it is used to model the total service time of an agent in a system; in insurance, it is used to model total claims on a number of policies in the individual risk model. In this talk, we focus on stochastic properties of linear combinations of heterogeneous random variables. We also discuss several nice applications in the diverse fields, especially in game theory.

A new variability order based on tail-heaviness

Alfonso Suárez-Llorens (alfonso.suarez@uca.es, University of Cádiz)
Miguel A. Sordo Díaz (mangel.sordo@uca.es, University of Cádiz)
Marilia C. de Souza (marilia.conceideso@mail.uca.es, University of Cádiz)

We introduce a new variability order that can be interpreted in terms of tail-heaviness which we will call the tail dispersive order. We provide the new definition, its interpretation and properties and the main characterization. We also study the relationship with other classical variability orders. Finally we study the tail dispersive order in some classical parametric families and provide some applications in insurance and finance. We conclude with a numerical example applied to log returns distributions.

Contributed talks

On univariate extreme value statistics and the estimation of the conditional tail expectation

Rassoul Abdelaziz (rsl_aziz@yahoo.fr, National School of Hydraulic Engineering of Blida)

In this paper, we study some estimations of the conditional tail expectation for the heavy-tailed distributions, in particular in the case when the second moment is infinite, our consideration are based on the study of the estimation of the first, second and third order parameters of the regular variation conditions about the distribution. Next to the construction of estimators, we also consider the corresponding asymptotic results and some illustrations by finite sample behaviour through a real insurance application as well as estimations.

References

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Two-sample Bayesian prediction intervals for progressively type-II censored competing risks data from the half-logistic distribution

Alaa H. Abdel-Hamid (hamid_alh@yahoo.com, Beni-Suef University)
Atef F. Hashem (dr_afh2010@yahoo.com, Beni-Suef University)

Two-sample scheme is used to predict the s-th order statistic in a future sample. The informative sample is assumed to be drawn from a general class of distributions which includes, among others, Weibull, compound Weibull, Pareto, Gompertz and half-logistic distributions. The informative and future samples are progressively type-II censored, under competing risks model, and assumed to be obtained from the same population. A special attention is paid to the half-logistic distribution. Using three different progressive censoring schemes, numerical computations are carried out to illustrate the performance of the procedure. An illustrative example based on real data is also considered. The coverage probabilities and average interval lengths of the prediction intervals are computed via a simulation study.

Bayes estimation of the Pareto parameters under progressive censoring data for constant-partially accelerated life tests using MCMC

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Accelerated life testing (ALT) is widely used in product life testing experiments since it provides significant reduction in time and cost of testing. In this paper, assuming that the lifetime of items under use condition follow the two-parameter Pareto distribution of the second kind, partially accelerated life tests (PALTs) based on progressively Type-II censored samples are considered. The likelihood equations of the model parameters and the acceleration factor are reduced to a single nonlinear equation to be solved numerically to obtain the maximum likelihood estimates (MLEs). The classical Bayes estimates cannot be obtained in explicit form, so, we propose to apply Markov chain Monte Carlo (MCMC) to tackle this problem, which allows us to construct the credible interval of the involved parameters. Analysis of a simulated data set has also been presented for illustrative purposes. Finally, a Monte Carlo simulation study is carried out to investigate the precision of the Bayes estimates with MLEs.

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Class of estimator for finite population variance under two-phase sampling

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Arcos and Rueda (1997) and Ahmed *et al.* (2000) suggested some estimators for finite population variance using multiple auxiliary variables. Al-Jararha and Ahmed

(2002) suggested a class of estimators for finite population variance using two auxiliary variables under two-phase sampling. In this paper, we have proposed a class of estimators for finite population variance using multiple auxiliary variables under two-phase sampling. The properties of this class has been studied under simple random sampling without replacement.

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Estimation of the reliability in the stress-strength models under the modifications of the ranked set sampling: Weibull distribution

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Akgül and Şenoğlu (2014) obtained ML and MML estimators of the system reliability $R = P(X < Y)$ using ranked set sampling (RSS) when the distributions of both the stress X and the strength Y are Weibull, see also Akgül (2014). They assume that X and Y are independent. See also McIntre (1952) and Kotz et al. (2003) in the context of RSS method and the stress-strength models, respectively. They compared the efficiencies of these estimators with the corresponding ML and MML estimators of R based on simple random sampling (SRS) data. Simulation results show that the proposed estimators based on RSS are more preferable than the estimators based on SRS.

Besides being more efficient, RSS is also cost effective method. Therefore, there have been various attempts to obtain modifications of it, such as extreme ranked set sampling (ERSS) (see, Samavi et al. 1996), median ranked set sampling (MRSS) (see, Muttlak 1997) and percentile ranked set sampling (PRSS) (see, Muttlak 2003), in the last decades. Main advantage of these modifications is that they are useful to overcome ranking error encountered in RSS method.

In this study, we estimate the system reliability R in the stress-strength models under the ERSS, MRSS and PRSS sampling methods. Similar to Akgül and Şenoğlu (2014), we use the maximum likelihood (ML) and the modified maximum likelihood (MML) (see, Tiku 1967) estimation methods. We then compare the performances of the proposed estimators with the traditional estimator of R based on SRS via an extensive Monte Carlo simulation study. Bias and mean square error (MSE) criterions are used in the comparisons for determining the most efficient estimator and the sampling method.

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Meta-analysis based on minimal repair times of independent systems

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Consider $k \geq 1$ independent and identically structured systems, each with a certain number of observed repair times. The repair process is assumed to be performed according to a minimal-repair strategy. In this strategy, the state of the system after the repair is the same as it was immediately before the failure of the system. The resulting pooled sample is then used to develop parametric as well as exact nonparametric inferential procedures about the population. The distribution theory of the ordered polled sample of minimal repair times is developed in this work. The Best Linear Unbiased Estimators (BLUEs) as well as Best Linear Invariant Estimators (BLIEs) of the location and scale parameters of the presumed parametric families of life distributions are obtained. Furthermore, the Best Linear Unbiased Predictor (BLUP) and the Best Linear Invariant Predictor (BLIP) of a future repair time from an independent system are also obtained. Exact distribution free confidence intervals for quantiles of the population as well as exact prediction intervals for future record values are developed. It is observed that both parametric and nonparametric estimators based on the pooled sample are overall more efficient than those based on one sample of the same size and also than those based on independent samples. A real data set of Boeing air conditioners, consisting of successive failures of the air conditioning system of each member of a fleet of Boeing jet airplanes, is used to illustrate the inferential results developed here. Finally, the future works and open problems in this context is discussed.

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A class of measures of shape: properties and orderings

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The right tail behaviour of continuous probability distributions is an important issue in many areas of research. In this work, we consider a class of measures that capture some information about this behaviour and study its properties. We also give a characterization of scaling free probability distributions using this class. In particular, we study the relationship between the stochastic order induced by this class and the well-known convex transform order. This research is still in progress.

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A comparison of different methods for estimating the missing value in ANOVA

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Aydin and Şenoğlu (2014) obtained the estimates of the missing value in one-way ANOVA model by using the maximum likelihood (ML), modified maximum likelihood (MML) and the traditional least squares (LS) methods when the distribution of the error terms is nonnormal, i.e., long-tailed symmetric (LTS), see also Aydin (2013). In this study, we generalize their study to two-way ANOVA model. We obtain ML estimates of the missing value by using two different approaches. First, iteratively reweighting algorithm (IRA) which is very popular iteration-based method to obtain the maximum likelihood estimates of the model parameters when the likelihood equations have no explicit solutions was used, see Lawson (1961). It should be noted that IRA is an EM type algorithm, since LTS distribution can be written as a scale mixture of the normal distribution. Therefore, its convergence is guaranteed. Second, MML methodology which is a non-iterative method was used to obtain the explicit estimator of the missing value, see Tiku (1967). It is based on the idea of the linearization of the intractable terms in the likelihood equations by using the first two terms of Taylor series expansion around the expected values of the standardized order statistics. MML estimators are asymptotically equivalent to ML estimators besides being easy to compute. Monte Carlo simulation study was carried out to compare the efficiencies of the proposed estimators with the traditional LS estimator. Similar to Aydin and Şenoğlu (2014), ML was found to be the most efficient estimator followed by MML estimator as expected. Actually, efficiencies of the ML and the MML estimators are very close to each other as the theory says. At the end of the study, we analyzed a data set taken from the literature, see Ross (2009).

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Asymptotic distributions of order statistics and record values arising from the gamma and Kumaraswamy-generated-distributions families

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The asymptotic behavior of the order statistics and record values based on the gamma and Kumaraswamy-generated-distributions families are studied. In each case, the relation between the weak convergence of the base distribution and the generated family is revealed. Moreover the relations between the limit types of the base distribution and its generated family is studied.

Exact prediction intervals for future current records and record range from any continuous distribution

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In this paper, a general method for predicting future lower and upper current records and record range from any arbitrary continuous distribution is proposed. Two pivotal statistics with the same explicit distribution for lower and upper current records are developed to construct prediction intervals for future current records. In addition, prediction intervals for future observations of the record range are constructed. Moreover, simulation study is applied on normal and Weibull distributions to investigate the efficiency of the suggested method. Finally, an example for real lifetime data with unknown distribution is analyzed.

Two new maximum likelihood methods to estimate the sample size from the record values

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Records are defined in statistics as the consecutive maxima (or minima) of a sequence of random variables. These data are of interest in reliability, survival analysis, and fatigue tests where quite often only the record values are registered. There are also practical situations in which an observation is stored only if it is a record value, for instance in geosciences and sports. In this setting, our main interest is to estimate the sample size from the record values. So, we obtain several conditional distributions of record values given the number of records. We derive two likelihood functions, we show that they are unimodal and characterize the maximum likelihood estimators of n. In general, numerical techniques are necessary to find them. The results obtained are compared to estimators of n previously given in the literature. Their performances are explored through simulations. An application to a real dataset is also included.

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Optimal bounds on the bias of quasimidranges

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For given order statistics $X_{1:n} \leq \ldots \leq X_{n:n}$ of the sample $(X_1, \ldots, X_n)$ with common distribution function F and mean μ, and variance σ^2, the (r,s)–quasimidrange $M_{r,s}$ is defined as the arithmetic mean of rth and sth order statistics, i.e.

$$M_{r,s} = \frac{1}{2}\left(X_{r:n} + X_{s:n}\right), \quad 1 \leq r < s \leq n.$$

Using the projection method, we derive sharp upper and lower bounds on the bias $E(M_{r,s} - \mu)$ of estimation of unknown mean of the parent population by quasimidranges, expressed in standard deviation units. In this case the projected function has two maxima, and to derive the shape of the projection we introduce and analyze two auxiliary functions which determine the corresponding projection uniquely. As a result of numerical computations we show that the smallest error is obtained if r is close to $n/4$, and s is close to $3n/4$.

Type-I censored generalized order statistics

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In the talk, the distribution of Type-I censored generalized order statistics is examined. In the particular case of an underlying exponential distribution, conditional distributions of the corresponding spacings are given. Based on the Type-I censored sample of generalized order statistics, the distribution of the maximum likelihood estimator for the parameter of the exponential distribution is obtained.

Fuzzy-weighted ranked set sampling: a new way to construct ranked set sampling

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The concept of fuzzy sets and approximate reasoning have been used in many areas of different kinds of science because the uncertainty or fuzziness are already exist in every cases of real life. In this study we proposed a new approach for Ranked Set Sampling (RSS) procedure by using fuzzy set theory. The ranker and ranking mechanism is one of the major parts of RSS procedure. In RSS, personal knowledge or auxiliary variable are used for ranking. The ranking decision could not be always perfect because ranking is done without actual measurement of response variable. Therefore, there occurs a certain amount of uncertainty. We propose that the fuzzy theory could be a right way of dealing with that uncertainty. This procedure combines fuzzy sets concept and RSS procedure. A simulation study is constructed and the results show that our new procedure could be a remarkable way to construct RSS method.

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Inference on $P(Y < X)$ based on record values for generalized Pareto distribution

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In this paper we consider the problem of estimation of $R = P(Y < X)$ based on record values, when X and Y are two independent generalized Pareto distribution. The maximum likelihood estimators and confidence interval for R are obtained. The Bayes estimation of R is also considered in this work. Finally a simulation study is performed to obtain the efficiencies of different estimators proposed in this work.

Discrete q-uniform distribution

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Suppose that balls are successively drawn one after the other from an urn, initially containing one white and one black ball, according to the following scheme. After each drawing the drawn ball is placed back in the urn together with another ball of the same color. Assume that the conditional probability of drawing a white ball at the ith drawing, given that $j-1$ white balls are drawn in the previous $i-1$ drawings, varies geometrically, with rate q, both with the number of white balls and the total number of balls in the urn. The distribution of the number X_n of white balls drawn in n drawings, which turn out to be a q-analog of a Discrete Uniform distribution, may be called Discrete q-Uniform distribution. The q-factorial and the usual factorial moments of X_n are derived. The Discrete q-Uniform distribution is obtained as the congruence class, modulo n, distribution of Bernoulli generated numbers. Further, the Discrete q-Uniform distribution is deduced as the conditional distribution of a Geometric distribution, given its sum with another Geometric distribution independent of it. Also, the joint and the marginal distributions of the minimum and the maximum of a random sample of size m from a Discrete q-Uniform distribution, are derived and the distribution of the range is obtained.

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Nonparametric statistical intervals based on ordered data

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Some recent developments in constructing nonparametric statistical intervals based on various kinds of ordered (pooled) data are presented. This includes (progressively) Type-II censored data as well as minimal repair times (record values). In the talk, confidence, prediction, and tolerance intervals are discussed.

A test of multidimensional stochastic dominance of I order using multidimensional quantile functions

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Let there be a probability distribution on $\mathbb{R}^d$ with a cumulative distribution function G and a sample $X_1, X_2, \ldots, X_n \in \mathbb{R}^d$ from a probability distribution on $\mathbb{R}^d$ with a cumulative distribution function F. Consider the following problem of testing:

$$H : \neg(F \preceq G) \quad \vee \quad F = G \qquad \text{vs} \qquad K : F \preceq G \quad \wedge \quad F \neq G$$

where $F \preceq G$ means that $F(x) \leq G(x)$ for all $x \in \mathbb{R}^d$.

In the case of $d = 1$ many solutions of this problem are known, e.g. the one-side Kolmogorov-Smirnov test. Its construction is based on the fact, that under the hypothesis H a distribution of a statistics $\sup_{x \in \mathbb{R}}(F_n(x) - G(x))$, where F_n denotes an empirical distribution function based on the sample $X_1, X_2, \ldots, X_n$, doesn't depend on F (also in a limit with $n \to \infty$).

In the case of $d > 1$ the property above doesn't hold, but there are some solutions of the problem in the literature, e.g based on the same statistics as in the case of $d = 1$ and on estimation its distribution under H with some simulations.

In the talk another test will be proposed, which is based on multidimensional quantile functions of Einmahl and Mason. Under H the distribution of its statistics doesn't depend on F.

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Application of ranked set sampling on correlation coefficient (R) test for normality

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Normality assumptions are used in many statistical analyses and applications of statistics. Thus there are many tests to check the normality assumption. One of the most primitive ones is the Correlation Coefficient (R) test for normality. Although it is a primitive test, due to its ease of application it is commonly used and preferred. In this study, the aim is to improve the Correlation Coefficient (R) test for normality by using the power of ranked set sampling. It is known that ranked set sampling, in general, represents the population in a better way. By using simple random sampling for different distributions the power of Correlation Coefficient (R) test is calculated and these are compared with the results by using ranked set sampling. For different cycles (M=1, 3, 5) ranked set sampling is applied to compute critical values in the Correlation Coefficient (R) test. The results demonstrate improved power compared to the simple random sampling. This preliminary study shows that the use of ranked set sampling for normality tests should further be investigated in tests like the Z test, which was developed by M.L. Tiku.

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Characterizations based on moments of the number of observations near-order statistics

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Let $X_1,...,X_n$ be independent and identically distributed (iid) real-valued random variables with continuous distribution function F, and denote the corresponding ith order statistic by $X_{i:n}$. Two random variables $K_+(n,k,a) = \#\{i : X_i \in [X_{k:n}, X_{k:n} + a)\}$ and $K_-(n,k,a) = \#\{i : X_i \in (X_{k:n} - a, X_{k:n}]\}$ have been defined in the literature, where $k = 1,...,n$ and $a > 0$ is a constant. Several articles have been published to study asymptotic behavior of the aforementioned random variables, see for example Balakrishnan and Stepanov (2005) and Dembińska et al. (2007). We provide some characterization results of F based on moments of $K_+(n,k,a)$ and $K_-(n,k,a)$ using the concept of completeness (Higgins, 2004) and the method of general solution of the functional equation (Aczél, 1966).

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Stochastic processes with good memory

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We will show a method of construction of simple, discrete time, stochastic processes that either are k-Markovian (with $k \geq 2$), or their memories are not limited. As the basic framework for that subject we have chosen the following 'reliability with repair' problem. Consider a system which starts to operate as new, and after each failure it is repaired. We do not consider repair times, so only the (random) times T_j ($j = 1,2,...$) between the consecutive ($j-1$ th and j-th) failures are of our interest. We will investigate, as the basic stochastic process, the ordered sequence S_n, where $S_n = T_1 + \ldots + T_n$.

In a situation like this the usual assumption is Markovianity of the stochastic process S_n. However, in practical situations this assumption is not always realistic. The probability distribution of a given time to n-th failure S_n may also depend on realizations $s_{n-2}, s_{n-3}, \ldots$ of the random variables $S_{n-2}, S_{n-3}, \ldots$ respectively.

Our goal is then to define (for $n \geq 3$) the corresponding conditional probability distributions $G_n(s_n|s_{n-1}, s_{n-2}, \ldots, s_{n-k})$ for some fixed $k \geq 2$ so that the underlying stochastic process "becomes" k-Markovian. In order to determine the conditional probability distributions $G_n(s_n|s_{n-1}, s_{n-2}, \ldots, s_{n-k})$ we apply the *'method of parameter dependence"* presented, among others, in our recent paper.

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Optimal bounds on expectations of order statistics and spacings based on the ID distributions

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We present the optimal upper bounds for the expectations of order statistics and spacings based on independent and identically distributed samples from the increasing density (ID) distributions. The results are determined by use of the projections of elements of Hilbert spaces onto properly chosen convex cones. The bounds are expressed by means of the population mean and standard deviation.

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Estimation of the inverse Weibull distribution based on progressively censored data: comparative study

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In this study we consider statistical inferences about the unknown parameters of the Inverse Weibull distribution based on progressively type-II censoring using classical and Bayesian procedures. For classical procedures we propose using the maximum likelihood; the least squares methods and the approximate maximum likelihood estimators. The Bayes estimators are obtained based on both the symmetric and asymmetric (*Linex*, General Entropy and Precautionary) loss functions. There are no explicit forms for the Bayes estimators, therefore, we propose the Lindley's approximation method to compute the Bayes estimators. A comparison between these estimators is provided by using extensive simulation and three criteria, namely, Bias, mean squared error and Pitman nearness (PN) probability. It is concluded that the

approximate Bayes estimators outperform the classical estimators most of the time. Real life data example is provided to illustrate our proposed estimators.

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On a class of half-logistic generated distributions

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Given a standard half-logistic cumulative distribution function (CDF) H, a new class F of distributions can be generated by composing H with another function $\eta(x) = -\ln G(x)$, where $G(x)$ is an absolutely continuous CDF on $[0,\infty)$, such that $R(x) \equiv 1 - F(x) = H[\eta(x)]$ is a survival function on $[0,\infty)$. This new class is so broad that G includes the Weibull, exponential, Rayleigh, Burr type XII, compound exponential (Lomax), compound Rayleigh, Gomprtz, compound Gomprtz, half-logistic, half-normal, Lindeley, exponentiated distributions and any distribution with positive domain. The CDF F of the new class shall be called half-logistic generated G-distribution (HLGGD). Some properties of F are presented. A real data set is analyzed using the generated class of distributions which shows that the half-logistic generated Burr type XII distribution can be used quite effectively in analyzing real lifetime data. Estimates (ML and Bayes) are obtained and computed.

Inference under a type-I censoring scheme with self-determined threshold

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In this work we introduce a type-I censoring scheme where the threshold time is determined after observing a certain number of failures. By assuming that the underlying lifetime distribution depends on some parameter θ, we first discuss the asymptotics of its MLE, $\hat{\theta}$. It is shown that under standard regularity conditions $\hat{\theta}$ has an asymptotic normal distribution. Next, we consider the case of exponential lifetimes and develop exact inferential procedures for the location and scale parameters. In particular, we discuss unbiased estimation and construction of exact confidence intervals.

Properties of aging intensity and reversed aging intensity functions

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In this talk properties of two functions of reliability theory: aging intensity and reversed aging intensity, will be presented. Moreover, aging intensity and reversed aging intensity orders will be determined for some Weibull distributions.

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Inequalities for variances of order statistics coming from urn models

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We consider the drawing with and without replacement models from numerical populations. For order statistics based on the above drawing schemes, we provide sharp upper bounds for their variances, expressed in the single observation variance units. We also characterize the populations for which the bounds are attained.

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Preservation of the GTTT transform order under some reliability operations

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Let h be a nonnegative measurable function. The generalized total time on test (GTTT) transform of the distribution F with respect to the function h is defined as a function

$$H_F^{-1}(u;h) = \int_{-\infty}^{F^{-1}(u)} h(F(x))dx, \; 0 < u < 1,$$

provided the integral is well defined. A distribution F is said to be smaller than G in the GTTT transform order with respect to the function h if $H_F^{-1}(u;h) \leq H_G^{-1}(u;h)$ for $u \in (0,1)$. This generalization of the usual total time on test transform was introduced by Li and Shaked and it is related to well known orders, e.g. the excess wealth order.

The GTTT transform $H_F^{-1}(u;h)$ can be considered as a quantile function of some distribution $H_F(\cdot;h)$. Bartoszewicz and Benduch studied stochastic comparision of GTTT transforms. We complete their results with the GTTT transform order. Another operation which preserves the GTTT transform order is mixing exponential distributions. We use Bartoszewicz's and Skolimowska's studies. We also recall a result obtained by Shaked, Sordo and Suárez-Llorens.

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Scale parameter estimation under an order statistics prior

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For a scheme of s independent samples based on distribution functions out of a location-scale family, the joint order restricted estimation of the s scale parameters is considered. This data structure can be found, for example, in constant stress models within accelerated life testing or in biostatistics within studies of lifetime distributions or characteristics of interest under different levels of a prognostic factor, different therapies or doses. The samples may be possibly differently Type-II censored and of different sample sizes, often small. However, in small samples, the classical order restricted inference procedures will frequently end up with a couple of ties among parameter estimates. If the estimation should yield strictly ordered parameters, a Bayesian set-up is proposed, provided that some prior information is available. In order to estimate s distribution parameters, which, in the statistical model, are supposed to be arranged in strictly ascending order, the associated prior distribution is considered to be the joint distribution of s order statistics from some underlying absolutely continuous distribution function G. Aiming at presenting explicit results, G is chosen to be an extended truncated Erlang distribution (ETED), and identical numbers of observations are considered for all samples. Without imposing any further restrictions on the model, the multivariate posterior distribution is stated explicitly and turns out to be Weinmans multivariate exponential distribution (Kotz, Balakrishnan and Johnson, 2000). The Bayes estimators are then obtained analytically.

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Homogeneity-testing in multiparameter exponential families

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Based on s stochastically independent samples with underlying density functions of the same multiparameter exponential family, the homogeneity hypothesis of identical densities is considered. An approach of Garren (2000) is extended, who derived the asymptotics for Matusitas affinity for the above scenario and, based on that, suggested several test statistics in the case of two densities. A new statistical test is proposed based on Toussaint´s affinity, and its properties are examined, while allowing for the sample sizes to grow proportionally without assuming the limits of respective ratios to be equal to one. The usefulness of this approach is demonstrated via simulation studies. The findings are applied to the situation of s independent $(n-r+1)$-out-of-n systems modeled by sequential order statistics.

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Distribution of the rank of order statistics from bivariate sample

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In this talk, we consider a bivariate random sample $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ from an absolutely continuous distribution function $F(x, y)$. Suppose that (X_{n+1}, Y_{n+1}), (X_{n+2}, Y_{n+2}), ..., (X_{n+m}, Y_{n+m}); $m \geq 1$ be a new sample with an absolutely continuous distribution function $G(x, y)$ and independent from $(X_i, Y_i), i = 1, 2, ..., n$. Let η_1 indicates the total number of new X observations $X_{n+1}, X_{n+2}, ..., X_{n+m}$ which does not exceed a random threshold based on the $X_{r:n}$. Analogously, η_2 shows the total number of new observations $Y_{n+1}, Y_{n+2}, ..., Y_{n+m}$ which does not exceed $Y_{s:n}$. We derive the joint probability mass function of discrete random vector (η_1, η_2). Some numerical results are also provided.

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Bootstrap method for central and intermediate order statistics under power normalization

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It has been known for a long time that for bootstrapping the distribution of the extremes under the traditional linear normalization of a sample consistently, the bootstrap sample size needs to be of smaller order than the original sample size. In this paper, we show that the same is true if we use the bootstrap for estimating a central, or an intermediate quantile under power normalization. A simulation study illustrates and corroborates theoretical results.

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Limit processes for sequences of partial sums of residuals of regressions against order statistics with Markov-modulated noise

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We consider following regression models: $Y_{ni} = \theta\xi_{i:n} + \varepsilon_i^{V_i}$, $n \geq 1$, $i = 1,\dots,n$ (the first model); $Y_{ni} = a + b\xi_{i:n} + \varepsilon_i^{V_i}$, $n \geq 1, i = 1,\dots,n$ (the second one). Here $\{\varepsilon_i^v, i \geq 1, 1 \leq v \leq M\}$ is a family of independent random variables where $\{\varepsilon_i^v, i \geq 1\}$ are identically distributed for each v, $\mathbf{E}\varepsilon_1^v = 0$, $\mathbf{D}\varepsilon_1^v = \sigma_v^2 \geq 0$ and $\sum_{v=1}^M \sigma_v^2 > 0$; $\{V_i\}_{i=1}^\infty$ is an irreducible aperiodic Markov chain on the state space $\{1,\dots,M\}$ with stationary distribution $\{\pi_i\}_{i=1}^M$; $\{\varepsilon_i^v, i \geq 1\}$, $\{V_i\}_{i=1}^\infty$ and i.i.d. $\{\xi_i\}_{i=1}^\infty$ are mutually independent. Let ξ_1 have distribution function F and finite positive $\mathbf{E}\xi_1^2$; for the second model $\mathbf{Var}\xi_1 > 0$ additionally.

Introduce *fitted values* $\{\widehat{Y_i}\}$, *regression residuals* $\{\widehat{\varepsilon_i}\}$ and their *partial sums* $\{\widehat{\Delta}_i\}$. An *empirical bridge* is a random polygon $\widehat{Z}_n$ with nodes $\left(k/n,\ (\widehat{\Delta}_k - k\widehat{\Delta}_n/n)/\sqrt{n\widehat{\sigma^2}}\right)$ where $\widehat{\sigma^2} = \overline{\widehat{\varepsilon}^2} - (\overline{\widehat{\varepsilon}})^2$ is an estimator of variance $\sigma^2 = \sum_{v=1}^M \sigma_v^2 \pi_v$.

Let $GL_F(t) = \int\limits_0^t F^{-1}(s)\,ds$ be the theoretical general Lorenz curve. Let $GL_F^0(t) = GL_F(t) - tGL_F(1)$ be its centered version.

Theorem *The empirical bridge $\widehat{Z}_n$ converge weakly, as $n \to \infty$, to the centered Gaussian process Z_F with covariance kernel, $K_F(t,s)$, given by*

$$K_F(t,s) = \min\{t,s\} - ts - \frac{GL_F^0(t)GL_F^0(s)}{\mathbf{E}\xi_1^2}, \ t,s \in [0,1]$$

for the first model,

$$K_F(t,s) = \min\{t,s\} - ts - \frac{GL_F^0(t)GL_F^0(s)}{\mathbf{Var}\xi_1}, \ t,s \in [0,1]$$

for the second one. Here weak convergence holds in the space $C(0,1)$ of continuous functions on [0,1] endowed with the uniform metric.

Minimum volume confidence regions in models of sequential order statistics with conditional proportional hazard rates

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As a particular structure in reliability, a sequential k-out-of-n system fails if more than $n-k$ of its n components fail, where the failure of some component may affect the residual lifetimes of the remaining components of the system. Sequential order statistics serve as a model for the (ordered) lifetimes of the components and the system, respectively. We focus on a particular sequential order statistic model, where the system's components are presumed to have conditional proportional hazard rates. In the situation of a pre-fixed baseline distribution, the model parameters are assumed to be unknown and have to be estimated from data. Confidence intervals for single model parameters as well as joint confidence regions for vectors of model parameters with different eligible properties are shown. Particularly, a minimum volume confidence region is presented. A simulation study is performed to illustrate the confidence sets and compare them regarding to volume and coverage probability of false parameters. Several other examples are shown.

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Best constant-stress accelerated life-test plans for one-shot devices under budget and time constraints

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We discuss here the design of constant-stress accelerated life-tests for one-shot devices by assuming Weibull distribution with non-constant scale and shape parameters as a lifetime model. Since there are no explicit expressions for the maximum likelihood estimators of the model parameters and their variances, we adopt the asymptotic approach here to develop an algorithm for the determination of optimal settings of allocation of devices, inspection frequency, and the number of inspections at each stress level. The asymptotic variance of the estimate of reliability of the device at a specified mission time is minimized subject to a pre-fixed experimental budget and a termination time. Examples are provided to illustrate the proposed algorithm for the determination of the best test plan. A sensitivity analysis of the best test plan is also carried out to examine the effect of misspecification of the model parameters.

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($\delta \geq 0$)-records

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A δ-record of a sequence of random variables is an observation that exceeds the current record by δ units. In the case $\delta \geq 0$, we present the distribution theory of δ-record values from an iid sample from an absolutely continuous parent. The particular case of exponential parents is studied with detail. We also point out some applications to type II counters, blocks in automobile traffic and queuing theory. This work complements some earlier results, see *Test*, **22** (4), 715–738, where the case $\delta \leq 0$ was considered.

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Sufficient conditions for the total time on test transform order

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This talk is devoted to the study of total time on test transform order. First we provide a characterization which reveals the true nature of this comparison. Given that this comparison is of particular interest when the stochastic order does not hold, we also provide sufficient conditions for the comparisons of the total time on test transforms, when the stochastic order does not hold. Applications to the comparison of several parametric families of distributions are provided.

Pólya-Aeppli shocks

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In these notes we consider a Pólya-Aeppli sequence of exponentially distributed random variables, see Chukova and Minkova (2013) and Minkova (2004). For this sequence we define the shocks as upper record values. We derive the distribution of the record values in a special case of non-identically distributed variables and apply it to the defined sequence. Then we derive the distribution of the record moments, the number of records and the number of zeros.

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Bounds on dispersion measures in Bayesian mixture models

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We present optimal bounds on quotients of various dispersion measures (variance, median absolute deviation, etc.) of general Bayesian mixtures. Our result can be applied in reliability theory – for example to find optimal bounds on lifetime variance of system with exchangeable elements in the units of lifetime variance of single element. It should be useful far beyond reliability theory as well.

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On concomitants of record values from generalized Farlie-Gumbel-Morgenstern distribution

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In this paper, we derive the distributions of concomitants of record values from generalized Farlie-Gumbel-Morgenstern family of bivariate distributions. We derive the single and the product moments of the concomitants for the general case. The results are then applied to the case of the two-parameters exponential marginal distributions. Using concomitants of record values we derive the best linear unbiased estimators of parameters of the marginal distributions. Moreover, two methods for obtaining predictors of concomitants of record values are presented. Finally, a numerical illustration is performed to highlight the theoretical results obtained.

Comparing relative skewness of random vectors

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The study of random variables or vectors usually involves the analysis of skewness. Measuring skewness via single quantities is to find a suitable stochastic order which captures the essence of what "F is less skewed than G" means. In this sense, van Zwet (1964) proposed a convex transform order for comparing skewness of two univariate distribution. In the literature, different extensions for multivariate distributions have been introduced. In this talk, we propose and analyze a new multivariate convex tranform order based on the standard construction. Properties and applications are discussed too.

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Spacings around an order statistic

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We determine the joint limiting distribution of adjacent spacings around a central, intermediate, or an extreme order statistic $X_{k:n}$ of a random sample of size n from a continuous distribution F. For central and intermediate cases, normalized spacings in the left and right neighborhoods are asymptotically i.i.d. exponential

random variables. The associated independent Poisson arrival processes are independent of $X_{k:n}$. For an extreme $X_{k:n}$, the asymptotic independence property of spacings fails for F in the domain of attraction of Fréchet and Weibull ($\alpha \neq 1$) distributions. This work also provides additional insight into the limiting distribution for the number of observations around $X_{k:n}$ for all three cases.

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Statistical inference for component distribution from system lifetime data

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In this talk, statistical inference of the reliability characteristics of the components in the system based on the lifetimes of systems with the same structure will be discussed. Both systems with independent and dependent components will be considered. Point and interval estimation methods for the parameters in different models are proposed. Monte Carlo simulation study is used to compare the performance of these estimation methods and recommendations are made based on these results. Examples of some two- and three-component systems are presented for illustrative purposes. Some recent developments on comparison of component lifetime distribution from system failure data will also be discussed.

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Association of zero-heavy continuous variables

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Zero-inflated continuous distributions can be seen as mixtures of a distribution concentrated at zero and a continuous distribution. They are used for modeling data that can be often encountered in research areas such as health, environment, or insurance. Classical estimator of Kendall's τ association index, originally designed for continuous data, should not be applied to zero-heavy data as it becomes biased and therefore lowers coverage probability of corresponding confidence intervals. We propose an estimator of τ that coincides with the classical one in the case of no excess of zero values, but has smaller bias and better coverage probability when multiple zero values are present in the data. Properties of both estimators, a classical and a proposed one, are compared in an extensive simulation study.

Prediction for future exponential lifetime based on random number of generalized order statistics under a general set-up

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In this paper we develop two pivotal quantities to construct predication intervals for future exponential lifetime based on a random number of lower generalized order statistics (gOs) under a general set-up including progressive type II censored order statistics (pOs) with general scheme. Moreover, maximum likelihood predictor (MLP) for these future exponential lifetimes are derived. Finally, a simulation study illustrates and corroborates the theoretical results.

Stochastic monotonicity of estimators

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Let $\mathbf{X} = (X_1, \ldots, X_n)$ be a sample from distribution with density $f(x;\theta)$, $\theta \in \Theta \subset \mathcal{R}$. In this talk the estimation of the parameter θ is considered. There are given conditions under which the estimators of the parameter θ have the property of preserving stochastic orders. Various types of stochastic orders are considered.

Bounds on expectations of L-statistics from maximally and minimally stable samples

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The random variables $X_1, X_2, \dots, X_n$ are said to be maximally (resp. minimally) stable of order j ($j = 1, 2, \dots, n$) if the distribution $F_{(j)}$ of $\max\{X_{k_1}, \dots, X_{k_j}\}$ (resp. $G_{(j)}$ of $\min\{X_{k_1}, \dots, X_{k_j}\}$) is the same for any j element subset $\{k_1, \dots, k_j\}$ of $\{1, 2, \dots, n\}$ (see Papadatos, 2001). Under the assumption of maximal (resp. minimal) stability of order j, we give lower and upper bounds on linear combinations $\sum_{k=j}^{n} c_k P(X_{k:n} \le x)$ (resp. $\sum_{k=1}^{n-j+1} d_k P(X_{k:n} > x)$) in terms of $F_{(j)}$ (resp. $G_{(j)}$), and present the corresponding sharp expectation bounds. Our results extend those of Papadatos (2001) to the case of L-statistics. Moreover, we derive some attainable Bonferroni-type inequalities.

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Maximizing the expected range from dependent samples under mean-variance information

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We derive the best possible upper bound for the expected range when the observations of the sample are possibly dependent and with possibly different marginal distributions, provided we know their means and variances. The proposed upper bound is best possible: for given means and given (strictly positive) variances, we construct a random vector satisfying the moment restrictions, and such that its expected range attains the bound. Also, we characterize the maximizing random vectors.

Our technique requires a novel (bivariate) adaption of an optimization method utilized by Bertsimas, Natarajan and Teo (2006); this method was effectively ap-

plied in the derivation of tight upper bounds for the sample maximum under mean-variance information. We compare our results with the classical AG bound for the expected range, obtained by Arnold and Groeneveld (1979). We also provide necessary and sufficient conditions for the AG bound to be tight.

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Estimation of harmonic and geometric mean in ranked set sampling

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In this study, we develop an estimator for harmonic and geometric mean of population in ranked set sampling design. Ranked set sampling is applicable sampling design by a judgment method or based on the measurement of an auxiliary variable on the unit selected. The harmonic and geometric mean have numerous engineering applications, for instance: design streamflow estimation for wasteload application areas, and estimation of effective petrochemical and geophysical properties of a heterogeneous system of porous media. The study investigates the bias and relative efficiency of the proposed mean estimators. The efficiency comparison is made for some distributions. It is shown that the new estimators out performs its competitors in the literature.

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Stochastic comparisons of some conditional distributions under dependence

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Consider a portfolio of n individual risks or losses $X_1,...,X_n$ and assume that the impact of a possible dependence among the individual risks is modeled by a random vector $\mathbf{X} = (X_1,...,X_n)$ with some dependence structure. Let F_i be the distribution function of X_i and let F_i^{-1} be the corresponding quantile function, defined by $F_i^{-1}(p) = \inf\{x : F_i(x) \geq p\}$, $0 \leq p \leq 1$. Conditional distributions of the form

$$\left\{X_i \,\middle|\, \psi(\mathbf{X}) > F_\psi^{-1}(p)\right\} \tag{1}$$

where $p \in (0,1)$ and ψ is an increasing transformation from $\mathbf{R}^n$ to $\mathbf{R}$ with distribution function F_ψ and

$$\left\{X_i \,\middle|\, \bigcap_{j\neq i}^{n} \left\{X_j > F_j^{-1}(p_j)\right\}\right\} \tag{2}$$

where $p_j \in (0,1)$ for $j = 1,...,n$, describe the marginal i-th risk under adverse events. These conditional distributions have physical interpretations in insurance and finance. For example, by taking $\psi(\mathbf{X}) = X_1 + ... + X_n$, the distribution (1) describes the marginal contribution of the i-th risk to the aggregate risk. If we take $\psi(\mathbf{X}) = X_{(n)}$, where $X_{(n)} = \max\{X_1,...,X_n\}$, then (1) represents the size of the i-th risk given that the maximum risk exceed its value at risk for a given confidence level. Similar interpretations can be made by taking $\psi(\mathbf{X}) = X_{(1)}$, where $X_{(1)} = \min\{X_1,...,X_n\}$, and more generally $\psi(\mathbf{X}) = X_{(j)}$, where $X_{(j)}$ is the j-th order statistic. The conditional distribution (2) describes the size of the i-th risk given that all the other risks exceed their respective values at risk for given confidence levels.

In this talk, we provide stochastic bounds and comparison results involving conditional distributions of the form (1) and (2).

Sequential estimation of a common location parameter of two populations

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The problem of sequentially estimating a common location parameter of two independent populations from the same distibution with an unknown location parameter and known but different scale parameters is considered in the case when the observations become available at random times. Certain classes of sequential estimation procedures are derived under a location invariant loss function and with the observation cost determined by convex functions of the stopping time and the number of observations up to that time.

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Exceedance-type tests

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Suppose X and Y are random variables with continuous univariate distributions F and G, respectively. For testing the hypothesis $H_0:\ F(x) = G(x)$ against the alternative $H_A:\ F(x) \geq G(x)$, there are simple tests based available on precedences and exceedances. One can count the number of observations in the Y-sample above all observations in the X-sample, or the number of observations in the X-sample below all those in the Y-sample. As suggested by Tukey, one or both of these statistics might be used to test H_0 against H_A. The test based on the sum of these two quantities is mentioned as the earliest work of Šidák on nonparametric statistics.

The extreme sample values may get inflated by possible outliers, which may adversely affect these test statistics. For this reason, we may want to reduce their influence by defining thresholds above the smallest and below the largest observed values in the samples. Let $X_1,\ldots,X_m$ and $Y_1,\ldots,Y_n$ be two independent random samples from the distributions F and G, respectively and denote the ordered X's and Y's by $X_{(1)} < \cdots < X_{(m)}$, and $Y_{(1)} < \cdots < Y_{(n)}$, respectively. Thresholds based on $(r+1)$-th order statistic from the Y-sample and $(m-s)$-th order statistic from the X-sample define the exceedance and precedence statistics of the form

$$\begin{aligned} A_s &= \text{the number of } Y\text{-observations larger than } X_{(m-s)}, \\ B_r &= \text{the number of } X\text{-observations smaller than } Y_{(1+r)}, \end{aligned}$$

where $0 \leq s < m$ and $0 \leq r < n$.

We study a family of rank statistics for the two-sample problem in which the test statistic is a sum of A_s and B_r for appropriate choices of r and s. It includes the Šidák's test as a special case.

Characterizations of Weibull distributions through selected functions of reliability theory

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In this talk some Weibull distributions and inverse Weibull distributions will be characterized through selected functions of reliability theory: failure rate, aging intensity and reversed aging intensity.

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On parallel systems with heterogeneous Weibull components

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When the observations follow the exponential distribution with different scale parameters, there is an extensive literature on stochastic orderings among order statistics and spacings, see for instance two review papers by Kochar (2012) and Balakrishnan and Zhao (2013) on this topic. A natural way to extend these works is to consider random variables with Weibull distributions since it includes exponential distributions. In the presentation a short overview on the wide field of stochastic orderings is given, showing some results given by Torrado and Kochar (2014) and also some of the current research the author is doing in moment.

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Linearity of regression for overlapping order statistics

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We consider a problem of characterization of continuous distributions for which linearity of regression of overlapping order statistics, $\mathbb{E}(X_{i:m}|X_{j:n}) = aX_{j:n} + b$, $m \leq n$, holds. Such characterizations in the case $m = n$ have a long history going back to Fisz (1958). Due to a new representation of conditional expectation $\mathbb{E}(X_{i:m}|X_{j:n})$ as a linear combination of conditional expectations $\mathbb{E}(X_{l:n}|X_{j:n})$, $l = i, \ldots, n-m+i$, we are able to use the already known approach (see Dembińska and Wesołowski (1998)) based on the Rao-Shanbhag version of the Cauchy integrated functional equation. However this is possible only if $j \leq i$ or $j \geq n-m+i$. The case $i = j = 1$ has been studied in Ahsanullah and Nevzerov (2001). In the remaining cases (except the situation $i = m = 1$ considered in Wesołowski and Gupta (2001)) the problem remains open.

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Characterization of exponential distribution through record values and associated beta distribution

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It would be shown that the exponential distribution is the only one among a large class of continuous distributions, which satisfies a regression equation. This equation involves the regression function of a fixed record value given two other record values, one of them being previous and the other next to the fixed record value. It will be proved that the underlying distribution is exponential if and only if the above regression equals the expected value of an appropriately defined beta distributed random variable. The results are refinements and extensions of those in the reference below.

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A note on the cumulative residual entropy

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Entropy of Equilibrium distribution or Cumulative Residual Entropy (CRE) and its dynamic version have attracted attention of many researchers in the resent years. Rao et al (2004) introduced CRE for a non-negative random variable in terms of cumulative distribution function as

$$\varepsilon(X) = -\int_0^\infty \overline{F}(x) \log \overline{F}(x) dx,$$

CRE has many interesting applications in different branches of sciences such as reliability theory, computer vision, image processing and etc. Asadi and Zohrevand (2007) showed that CRE is equal to expectation of mean residual life function and introduced the dynamic version of CRE (DCRE). They studied some characterization properties of DCRE.

Several properties and applications of CRE and DCRE have studied by many researchers such as Wang and Vemuri (2008), Navarro et al (2010), Di Crescenzo and Longobardi (2009), (2011). In this paper, we study the estimation problem of CRE for sample data. Also estimator of CRE for both complete and non-complete data are presented. In following, we argue about the distribution of estimator of CRE in non-censored samples of exponential distribution.

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