Abstracts

Convergence in law for the minimum of a branching random walk

Elie Aidekon

We consider a branching random walk on the real line. At each integer time, particles make independent steps then split. We are interested in the position of the leftmost particle of this population as time goes to infinity. We show that this minimum, once recentered around its mean, converges in law. Our proof gives a description of the trajectory of the whole path of the leftmost particle as well. This is the analog of the well-known result of Bramson in the setting of the branching Brownian motion.

Quicksort: a paradigm for the analysis of random recursive structures with branching - Baby talk

Gerold Alsmeyer

Quicksort, developed by Hoare [1,2], is the presumably most widely used algorithm to sort lists of distinct numbers. Its standard version picks an element x from the list and then creates two sublists consisting of those numbers which are smaller, respectively larger than x. The same procedure is next applied to the sublists and the algorithm continues until all sublists are singletons. Complexity is usually measured in terms of the number of necessary comparisons which becomes a random variable X_n when the input is chosen at random from the set of all lists of length n, w.l.o.g. a random permutation of $\{1, \ldots, n\}$. Rösler [3] has shown that $n^{-1}(X_n - \mathbb{E}X_n)$ converges in distribution to a limit law which solves a certain stochastic fixed-point equation (and is not Gaussian!). Possibly more interesting than the result itself, at least for mathematicians, is the way to derive it and I will try to provide the main steps of the necessary analysis, a major ingredient being a contraction argument. This is also interesting because it may serve as a template for the approach to many other problems concerning random recursive structures with inherent branching.

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The stationary tail index of contractive iterated function systems

Gerold Alsmeyer

Let $(Xn)_{n\geq 0}$ be a contractive iterated function system (IFS) on a complete separable metric space $(\mathbb{X}, d)$ with unbounded metric d, i.e.

$$X_n = \Psi_n \circ \ldots \circ \Psi_1(X_0)$$

for $n \geq 1$, where $\Psi_1, \Psi_2, \ldots$ are iid random Lipschitz functions on $\mathbb{X}$ with Lipschitz constants $L(\Psi_1), L(\Psi_2), \ldots$ Let π denote the unique stationary distribution of $(X_n)_{n\geq 0}$ and $x_0 \in \mathbb{X}$ an arbitrary reference point. Assuming $\mathbb{P}_\pi(d(x_0, X_0) > r) > 0$ for all $r > 0$, we will provide bounds for the lower and upper tail index ϑ_* and ϑ^* of $d(x_0, X_0)$ in equilibrium (under $\mathbb{P}_\pi$), defined by

$$\vartheta_* := -\limsup_{x\to\infty} \frac{\log \mathbb{P}(X > x)}{\log x} \qquad \text{and} \qquad \vartheta^* := -\liminf_{x\to\infty} \frac{\log \mathbb{P}(X > x)}{\log x}.$$

This will be done by providing lower and upper bounds for $d(x_0, X_n)$ under $\mathbb{P}_\pi$ in terms of rather simple IFS on $\mathbb{R}_\geq$ and the use of Goldie's implicit renewal theorem [2]. Special attention is paid to the particularly relevant case when $\mathbb{X} = \mathbb{R}$. The method is illustrated by some examples including the well-known AR(1) model with ARCH(1) errors which has been studied earlier in some detail by Borkovec and Klüppelberg [1].

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Revisiting Roe's theorem

Nils Byrial Andersen

Roe's theorem states that a function f on the real line with uniformly bounded derivatives and anti-derivatives must be a linear combination of the basic trigonometric functions. It can be generalized to weaker (polynomial) growth properties of the derivatives and anti-derivatives of f. The proofs usually rely on an argument to show that the support of the Fourier transform of f is inside the set $\{+/-1\}$, which in turn can be related to real Paley-Wiener theorems.

From restriction theorems to Strichartz inequalities - Baby talk

Jean–Philippe Anker

Strichartz inequalities are nowadays a fundamental tool in nonlinear PDE. They originate from restriction theorems for the Fourier transform. This lecture is meant as a survey for junior participants who are not familiar with this subject. Starting with the restriction theorems of Stein–Tomas and Strichartz, we shall move to dispersive and Strichartz inequalities for PDE.

The Hardy space $\mathbf{H^1}$ in the rational Dunkl setting

Jean–Philippe Anker & Jacek Dziubański

In the Euclidean setting, the Hardy space H^1 can be characterized in several ways, for instance by means of

- atoms,
- the heat maximal operator,
- the Poisson maximal operator,
- Riesz transforms.

We have extended these characterizations to the rational Dunkl setting, first in dimension one and next in the product case. At this occasion we have observed that the heat kernel fails to be Gaussian. Its decay is nevertheless sufficient in order to follow Uchiyama's approach. The general case would require sharp heat kernel estimates, which are not available for the time being.

In this series of two talks, we shall present our results, almost from scratch, including a multiplier theorem for H^1. Part of this work was done in collaboration with Néjib Ben Salem and Nabila Hamda (Faculté des Sciences de Tunis)

De Finetti-type results and discrete harmonic analysis

Dragu Atanasiu

Paul Ressel has proved, using results from discrete harmonic analysis, general theorems from which the theorem of Hewitt and Savage and other de Finetti-type results can be deduced. Our aim is to obtain, also using discrete harmonic analysis, more de Finetti-type results such as a generalization of Integrated Cauchy Functional Equation on a commutative semigroup.

Weighted norm inequalities for Weyl multipliers and some applications

Sayan Bagchi

In this talk we study weighted norm inequalities for Weyl multipliers satisfying Mauceri's condition. As applications of this we obtain some estimates for Lp multipliers on the Heisenberg group and also show in the context of a theorem of Weis on operator valued Fourier multipliers that the R-boundedness of the derivative of the multiplier is not necessary for the boundedness of the multiplier transform.

This is a joint work with my PhD supervisor, Professor Sundaram Thangavelu.

Estimates of semigroups of measures on the Heisenberg group

Krystian Bekała

We study semigroups of measures on the Heisenberg group. Such semigroups are characterized by their generators, which are tempered distributions. Assuming some weight and smooth conditions for Fourier transform of the generator we get pointwise estimates for densities of semigroups.

Weyl-Pedersen calculus for nilpotent Lie groups

Ingrid Beltiţă

The Weyl-Pedersen calculus is the correspondence $a \mapsto \mathrm{Op}^\pi(a)$ constructed by N.V. Pedersen in [Matrix coefficients and a Weyl correspondence for nilpotent Lie groups. *Invent. Math.* **118** (1994), no. 1, 1–36] as a generalization of the pseudo-differential Weyl calculus on $\mathbb{R}^n$. Here $\pi\colon G \to \mathbb{B}(\mathcal{H})$ is any unitary irreducible representation of the connected, simply connected, nilpotent Lie group G, while the symbol a is a tempered distribution on the coadjoint orbit $\mathcal{O}$ corresponding to π by the orbit method. The operator $\mathrm{Op}^\pi(a)$ is a linear operator in the representation space $\mathcal{H}$, and is in general unbounded.

We present here boundedness and compactness properties for the operators obtained by the Weyl-Pedersen calculus in the case of the irreducible unitary representations of nilpotent Lie groups that are associated with flat coadjoint orbits. We use spaces of smooth symbols satisfying appropriate growth conditions expressed in terms of invariant differential operators on the considered coadjoint orbit. On the other hand, we also give the construction of an inverse-close algebra of symbols, that are non necessarily smooth, corresponding to a inverse-closed closed algebra of bounded operators.

In the special case of the Schrödinger representation of the Heisenberg group we recover some classical properties of the pseudo-differential Weyl calculus, as the Calderón-Vaillancourt theorem and the Beals characterization in terms of commutators, respectively the Sjöstrand class of symbols.

This is joint work with Daniel Beltiţă (IMAR).

Asymptotic behavior of a convolution semigroup of Urbanik

Christian Berg

Let f be a non-zero Bernstein function, i.e., a function of the form

$$f(s) = a + bs + \int_0^\infty (1 - e^{-ts})\, d\nu(t), \quad a, b \geq 0,\ \nu \geq 0.$$

There exists a uniquely determined product convolution semigroup $(\rho_c)_{c>0}$ such that

$$\int_0^\infty x^n\, d\rho_c(x) = (f(1) \cdots f(n))^c, \quad c > 0, n = 0, 1, \ldots, \tag{1}$$

see [1]. The Stieltjes moment sequence in (1) is always determinate when $c \leq 2$ as an easy consequence of Carleman's criterion. However, for $c > 2$, it can be determinate or indeterminate depending on f. In fact, in the case $f(s) = s$, where the moment sequence is $(n!)^c$, it has been proved in [1] that the moment sequence is indeterminate. In this case $\rho_c = e_c(t)dm(t)$, where m is Lebesgue measure on the half-line and

$$e_c(t) = \frac{1}{2\pi} \int_{-\infty}^\infty t^{ix-1} \Gamma(1 - ix)^c\, dx, \quad t > 0. \tag{2}$$

As far as I know, the semigroup (e_c) has been considered first by Urbanik in [3], but he did not consider the indeterminacy question.

The proof of the indeterminacy in [1] is quite delicate based on asymptotic formulas for stable distributions due to Skorokhod.

In a joint paper with José Luis López, Spain, see [2], we have found the asymptotic behaviour of e_c at infinity, viz.

$$e_c(t) \sim \frac{(2\pi)^{(c-1)/2}}{\sqrt{c}} \frac{e^{-ct^{1/c}}}{t^{(c-1)/(2c)}}. \tag{3}$$

From (3) it is easy to derive the indeterminacy of e_c from a criterion of Krein.

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Martin kernel for fractional Laplacian in narrow cones

Krzysztof Bogdan

We give a power law for the homogeneity degree of the Martin kernel of the fractional Laplacian for the right circular cone when the angle of the cone tends to zero, thus resolving a decade-old puzzle. An application to the classical Laplacian in a complement of a plane slit by a cone is also given. This is a joint work with Bartłomiej Siudeja (University of Oregon) and Andrzej Stós (Université Blaise Pascal, Clermont-Ferrand).

Mixed Norm Estimates for the Riesz Transforms Associated to Dunkl Harmonic Oscillators

Pradeep Boggarapu

In this talk we study weighted mixed norm estimates for Riesz transforms associated to Dunkl harmonic oscillators. The idea is to show that the required inequalities are equivalent to certain vector valued inequalities for operator defined in terms of Laguerre expansions. In certain cases the main result can be deduced from the corresponding result for Hermite Riesz transforms. This is a joint work with prof. S. Thangavelu.

Sharp estimates of transition probability density for Bessel process in half-line

Kamil Bogus

In this talk we study the Bessel process $R_t^{(\mu)}$ with index $\mu \neq 0$ starting from $x > 0$. The transition probability density (with respect to the Lebesgue measure) of the process is expressed by the modified Bessel function of the first kind in the following way

$$p^{(\mu)}(t,x,y) = \frac{y}{t}\left(\frac{y}{x}\right)^{\mu} \exp\left(-\frac{x^2+y^2}{2t}\right) I_{|\mu|}\left(\frac{xy}{t}\right),$$

where $x, y > 0$ and $t > 0$.

The density function of transition probabilities of this process, killed when it reaches a positive level a, (where $x > a > 0$), is given by the Hunt formula:

$$p_a^{(\mu)}(t,x,y) = p^{(\mu)}(t,x,y) - \int_0^t p^{(\mu)}(t-s,1,y) q_{x,a}^{(\mu)}(s) ds,$$

where $q^{(\mu)}_{x,a}(s)$ is density of the first hitting time $T^{(\mu)}_a = \inf\{t > 0 : \; R^{(\mu)}_t = a\}$ (see [2]).

Our main result is given in the following sharp estimate:

$$\frac{p^{(\mu)}_a(t,x,y)}{p^{(\mu)}(t,x,y)} \overset{\mu}{\approx} \left(1 \wedge \frac{(x-a)(y-a)}{t}\right)\left(1 \vee \frac{t}{xy}\right), \quad x,y > a, \quad t > 0.$$

Here $f(t,x,y) \overset{\mu}{\approx} g(t,x,y)$ means that there exist positive constants $c^{(\mu)}_1$ and $c^{(\mu)}_2$ depending only on the index μ such that $c^{(\mu)}_1 \leq f/g \leq c^{(\mu)}_2$ for every $x, y > a$ and $t > 0$.

This talk is based on joint work with Jacek Małecki [1].

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On unbounded invariant measures of stochastic dynamical systems

Sara Brofferio

We consider stochastic dynamical systems $X_n = \Psi_n(X_{n-1})$, where Ψ_n are i.i.d. random continuous transformations of $\mathbf{R}$. We assume that $\Psi_n(x)$ behave asymptotically like $A_n x$, for some random positive number A_n. The main example is the stochastic affine recursion $X_n = A_n X_{n-1} + B_n$. Our aim is to describe invariant Radon measures of the process $\{X_n\}$ in the critical case, when $\mathbf{E} \log A = 0$. Under optimal assumptions, we prove that those measures behave at infinity like dx/x. In the proof we strongly use some properties of random walks on the affine group. The talk will be based on a joint paper with Dariusz Buraczewski.

Fluctuation Theory of Markov-Random-Walks

Fabian Buckmann

For examining Markov-modulated stochastic difference equations a solid background on the fluctuation theory of Markov-Random walks is needed. In the case of a MRW, where the underlying Markov Chain has a countable state space and is positive recurrent, the first insights of generalising classical random walk results is presented.

Hitting times of Bessel processes

Tomasz Byczkowski

We discuss results related to exit times of Bessel processes from a half-line. We present an explicit representation of the above-mentioned exit times and derive sharp estimates of its density. As applications we provide estimates of the Green function of a half-space for the hyperbolic Brownian motion and discuss the estimates of the heat kernel of Bessel proces. Results were obtained jointly with J. Małecki and M. Ryznar.

Large deviation estimates for the exceedance times of perpetuity sequences

Jeffrey Collamore

In a wide variety of problems in pure and applied probability, it is relevant to study the extremal events of a perpetuity sequence. Estimates for the stationary tail distribution of a perpetuity sequence have been developed in the seminal papers of Kesten (1973) and Goldie (1991). They have shown, in particular, that if $Y_n = B_1 + A_1 B_2 + \cdots + (A_1 \cdots A_{n-1}) B_n$, then

$$\mathbf{P}\left\{\sup_n Y_n > u\right\} \sim C u^{-\xi} \quad \text{as} \quad u \to \infty.$$

Analogous asymptotics also hold for $Y_\infty := \lim_{n\to\infty} Y_n$ and for some extensions of this model, i.e., more general random maps.

Recently, it was shown in Collamore and Vidyashankar (2013) that the above estimates can also be obtained by new techniques, borrowing ideas from large deviation theory. The objective of this talk is to extend these large deviation methods to a new type of problem; namely, the characterization of the first passage time of a perpetuity sequence. Letting $T_u := (\log u)^{-1} \inf\{n : Y_n > u\}$ denote the scaled first passage time, we develop sharp asymptotics for $\mathbf{P}\{T_u \in G\}$ which show, in particular, that this sequence satisfies the large deviation principle. We then extend these estimates to some other related processes, including the corresponding forward process described in Letac (1986). (Joint work with D. Buraczewski, E. Damek, and J. Zienkiewicz.)

Free groups in linear groups - Baby talk

Michael Cowling

An old observation of von Neumann led to the the "von Neumann conjecture" that every nonamenable locally compact group contains a discrete nonabelian free subgroup. A famous paper of Tits proves this for matrix groups over local fields. More recently, counterexamples have been found.

We consider a related problem, namely, if Γ is a "big" subgroup of a semisimple Lie group G, and x is an element of Γ that generates an infinite discrete subgroup, what can be said about the set of y in Γ such that x and y generate a nonabelian free group. It was shown by Poznansky that the set of such y is Zariski dense; we explain why the set of such y is "of probability 1".

The spectrum of the Laplacian on a four-dimensional Lie group

Michael Cowling

Let L be a Laplacian or sub-Laplacian on a Lie group G, and let $\sigma_p(L)$ be the spectrum of L acting as a densely-defined operator on $L^p(G)$. There are examples where the spectrum is discrete, or where it is a half-line, or where it is a parabolic region, and if we go outside the Lie group arena, then other shapes are possible. If we restrict attention to Lie groups, what kind of set can $\sigma_p(L)$ be? Christ and Müller considered an example where the spectrum included open sets, but they did not identify the spectrum. We shall show that it is a cone.

Heat kernel asymptotic for subordinated random walk

Wojciech Cygan
joint work with Alexander Bendikov and Bartosz Trojan

We consider a random walk S_n^{ψ} in $\mathbb{Z}^d$, obtained by subordinating a strongly aperiodic random walk with a finite support according to the concept of discrete subordination. The function ψ, which is the Laplace exponent of subordinator is assumed to be a complete Bernstein function such that its behaviour at zero is prescribed in the realm of regularly varying functions. We prove a strong version of Tauberian type theorem which allows us to investigate the asymptotic behaviour of the tails of subordinator. Finally, restricting the class of complete Bernstein functions ψ, we find the asymptotic of the transition kernel of subordinated random walk.

Discreteness of the spectrum of Schrödinger operators with non-negative matrix-valued potentials

Gian Maria Dall'Ara

We present three results giving sufficient and/or necessary conditions for discreteness of the spectrum of Schrödinger operators with non-negative matrix-valued potentials, i.e., operators acting on $\psi \in L^2(\mathbb{R}^n, \mathbb{C}^d)$ by the formula

$$H_V\psi := -\Delta\psi + V\psi,$$

where the potential V takes values in the set of non-negative Hermitian $d\times d$ matrices.

The first theorem provides a characterization of discreteness of the spectrum when the potential V is in a matrix-valued A_∞ class, thus extending a known result in the scalar case ($d = 1$). We also discuss a subtlety in the definition of the appropriate matrix-valued A_∞ class.

The second result is a sufficient condition for discreteness of the spectrum, which allows certain degenerate potentials, i.e., such that $\det(V) \equiv 0$. To formulate the condition, we introduce a notion of oscillation for subspace-valued mappings.

Our third and last result shows that if V is a 2×2 polynomial potential, then $-\Delta + V$ has discrete spectrum if and only if the scalar operator $-\Delta + \lambda$ has discrete spectrum, where $\lambda(x)$ is the minimal eigenvalue of $V(x)$.

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Perpetuities with slowly varying tails

Piotr Dyszewski

Consider a sequence of independent identically distributed two–dimensional random vectors $\{(A_n, B_n)\}_{n\in\mathbb{N}}$. Using this sequence we can define a Markov chain via the random difference equation

$$R_{n+1} = A_{n+1}R_n + B_{n+1} \qquad \text{for } n \geq 0,$$

where R_0 is arbitrary but independent of the sequence $\{(A_n, B_n)\}_{n\in\mathbb{N}}$. Put $(A, B) := (A_0, B_0)$. It is a well known fact that if $\mathbb{E}[\log|A|] < 0$ and $\mathbb{E}[\log^+|B|] < \infty$ the Markov chain $\{R_n\}_{n\in\mathbb{N}}$ has a unique stationary distribution which can be represented as the distribution of the random variable

$$R = \sum_{n\geq 0} B_{n+1} \prod_{k=1}^{n} A_k.$$

Random variable R is a solution of stochastic difference equation and appears, for example, in insurance mathematics, where it is called perpetuity. Assuming that $\log(|A| \vee |B|)$ is heavy-tailed we will find precise asymptotic of $\mathbb{P}[R > x]$ as $x \to \infty$.

Poisson boundary and Random walks

Behrang Forghani

Given a countable group G equipped with a probability measure μ, we will propose a machinery way to produce probability measures on group G with the same Poisson boundaries. Moreover, we will show how the asymptotic behaviors (for instance, asymptotic entropy) of random walks generated by these probability measures are related.

The lattice point counting problem on the Heisenberg groups

Rahul Garg

We consider the natural radial and Heisenberg-homogeneous norms on the Heisenberg groups given by $N_{\alpha,A}((z,t)) = \left(|z|^\alpha + A|t|^{\alpha/2}\right)^{1/\alpha}$, for $\alpha \geq 2$ and $A > 0$. This family includes the canonical Cygan-Korányi norm, corresponding to $\alpha = 4$. We study the lattice points counting problem on the Heisenberg groups, namely establish an error estimate for the number of points that the lattice of integral points has in a ball of large radius R. The exponent we establish for the error in the case $\alpha = 2$ is the best possible, in all dimensions. This is joint work with Amos Nevo and Krystal Taylor.

Solution to a semilinear elliptic problem in a Greenian domain

Zeineb Ghardalou

Let L be a second order elliptic operator with smooth coefficients satisfying $L1 = 0$ defined in a domain Ω that is Greenian for L. Under fairly general hypotheses on the function φ, we discuss existence, uniqueness and regularity of solution of the following problem:

$$\begin{cases} Lu + \varphi(.,u) = 0, & \text{in } \Omega; \\ u > 0, & \text{in } \Omega\ ; \\ u = 0, & \text{on } \partial\Omega. \end{cases}$$

Combinatorial Markov Chains

Rudolf Gruebel

By a combinatorial Markov chain we mean a Markov chain $X = (X_n)_{n\in\mathbb{N}}$ that is adapted to a combinatorial family $\mathbb{F}$ in the sense that

$$P(X_n \in \mathbb{F}_n) = 1, \quad P(X_n = y) > 0 \text{ for all } y \in \mathbb{F}_n,$$

where $\mathbb{F}_n$ denotes the set of combinatorial objects that have size n. Such processes typically arise in connection with sequential algorithms that transform an input sequence $\eta_1, \eta_2, \ldots$ into an output sequence $y_1, y_2, \ldots$ if $y_{n+1} \in \mathbb{F}_{n+1}$ depends on y_n and η_{n+1} only.

We review various boundary concepts for combinatorial Markov chains, with emphasis on the interplay with the Analysis of Algorithms: Boundaries may be used to obtain strong limit theorems for functionals of interest, but the existence of an algorithm generating X from i.i.d. input data can also be used to obtain the boundaries.

Quadratic maps in statistics

Piotr Graczyk

When X is a Gaussian random vector $N(m, \Sigma)$ with unknown mean m and covariance Σ and we dispose of a sample $X^{(1)}, X^{(2)}, ..., X^{(s)}$, the maximum likelihood estimators $\hat{m} = \bar{X}_n$ and $\hat{\Sigma}$ are well known. The estimator $\hat{\Sigma}$ is given by a quadratic map

$$\hat{\Sigma} = \frac{1}{s}\sum_{i=1}^{s}(X^{(i)} - \bar{X}_n)^t(X^{(i)} - \bar{X}_n)$$

and has a Wishart law on the symmetric cone $\mathrm{Sym}^+(n, R)$ of positive definite symmetric real matrices of size $n \times n$. The study of Wishart laws is based on harmonic analysis on symmetric cones, developed in the book of Faraut and Koranyi.

In modern statistics some relations of conditional independence between the components X_j of X must be considered. This leads to models of $N(m, \Sigma)$ such that the concentration matrix Σ^{-1} has some obligatory zeros. The position of these zeros is encoded by a graph $\mathcal{G}$. This is the starting point of the statistical graphical models theory, initiated in 1976 by Lauritzen. The harmonic analysis foundations of this theory are still unsatisfactory.

On the other hand, in statistical samples, even in the classical situation described in the beginning of this abstract, some data is sometimes missing. We propose quadratic maps useful in the estimation of the parameter Σ for monotonous missing data problem.

The talk is based on recent common research with H. Ishi(Nagoya) and S. Mamane(Johannesburgh).

Dirichlet heat kernel for unimodal Lévy processes

Tomasz Grzywny

We present some recent results about the transition density of jump-type isotropic unimodal Lévy processes X (i.e. rotation invariant Lévy process with the absolutely continuous Lévy measure which density is radially nonincreasing) killed upon leaving

D under scaling conditions at infinity for the Lévy-Kchintchine exponent of X. We derive sharp estimates of $p_D(t,x,y)$ for $C^{1,1}$ open sets D. The following factorization holds [2], [5]

$$p_D(t,x,y) \approx P^x(\tau_D > t)P^y(\tau_D > t)p(t \wedge T, x, y). \tag{4}$$

Here $P^y(\tau_D > t)$ is the survival probability of the corresponding process X, $p(t,x,y) = p_{\mathbb{R}^d}(t,x,y)$ is the (free) heat kernel for $\mathbb{R}^d$ and $T = T(D)$. Further we show estimates of $P^y(\tau_D > t)$ obtained in [2], [4] and estimates of $p(t,x,y)$ proved in [3].

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Hilbert transform along measurable vector fields constant on Lipschitz curves

Shaoming Guo

We prove the L^2 boundedness of the Hilbert transform (without cut-off) along a family of measurable vector fields which are constant on Lipschitz curves with a global angle condition, generalizing Bateman and Thiele's result on the one-variable vector fields.

Limit theorems for random processes with immigration at the epochs of a renewal process

Alexander Iksanov

I shall talk about random processes with immigration at the epochs of a renewal process. This class of random processes which alternatively may be called renewal shot-noise processes with random response functions includes perpetuities, branching processes with immigration, the number of busy servers in the GI/G/∞ queue and many others. I shall discuss two situations in which the processes, properly rescaled, converge weakly without normalization and centering. In the first case, there is the weak convergence in the Skorokhod space endowed with the J_1-topology, the limits being stationary processes with immigration. In the second case, there is the weak convergence of the finite-dimensional distributions only, the limits being conditionally Gaussian processes which take values in the Skorokhod space with probability less than one.

Poisson Shot Noise Lévy Model and Limit Theorems for Significant Functionals

Wissem Jedidi

In Internet traffic modeling, many authors presented models based on particular fractal shot noise representations. The inconvenience of these approaches is the multitude of assumptions and the lack of tools to check them. In this paper we propose a unified model based on a general Poisson shot noise representation for the cumulative input process (CIP). We present a procedure of approximation for this process; then we give a procedure for controlling the bandwidth of Internet providers. The approximation and control go via limit theorems for imporatnt functionals of the CIP, namely, the supremum process, the right inverse, and the storage mapping.

Boundary preserving transformations of random walks

Vadim Kaimanovich

I will describe a large class of transformations of random walks which preserve their Poisson boundary and the space of bounded harmonic functions. Joint work with B. Forghani.

Inversions in the flag kernels algebra. The Heisenberg group case

Grzegorz Kępa

Flag kernels are tempered distributions which generalize these of Caldéron-Zygmund type. For any homogeneous group $\mathbb{G}$ the class of operators which acts on $L^2(\mathbb{G})$ by co- involution with a flag kernel is closed under composition. In the case of the Heisenberg group we prove the inverse-closed property. It means that if an operator from this algebra is invertible then its inversion remains in the class.

Quantitative norm convergence of some ergodic averages

Vjekoslav Kovač

Multiple ergodic averages for several commuting transformations are known to converge in the L^2 norm. This fact was established gradually: by J. von Neumann for a single transformation, by J.-P. Conze and E. Lesigne for two commuting transformations, and finally by T. Tao for three or more commuting transformations. However, it is an interesting problem to quantify the convergence by estimating the norm-variation of the sequence of averages, or equivalently, to control the number of its jumps of size at least ε. In the linear case this was established by R. L. Jones, I. V. Ostrovskii, and J. M. Rosenblatt, while a very special instance of the bilinear case can be quantified using stronger pointwise estimates of J. Bourgain. We are able to go a bit further for bilinear and some trilinear averages, but for several technical reasons we need to work in a toy model: with commuting actions of a Cantor group, i.e.a direct sum of countably many copies of a fixed finite cyclic group. Related problems in this setting have already been considered by V. Bergelson, T. Tao, and T. Ziegler.

This is a joint work with Christoph Thiele (University of Bonn).

Special moduli of continuity and the constant in the Jackson-Stechkin theorem

Yuriy Kryakin

We consider a special $2k-$order modulus of continuity $W_{2k}(f,h)$ of $2\pi-$periodic continuous functions and prove the Jackson – Stechkin inequality for trigonometric polynomials with exact constants.

We also give an analog of the Bernstein – Nikolsky – Stechkin inequality in terms of W_{2k}. The inequality $W_{2k}(f,h) \leq 3\|f\|$ and the Bernstein – Nikolsky – Stechkin type estimate imply the Jackson – Stechkin theorem with nearly optimal constant for approximation by periodic splines.

Boundary Representations of the Free Group.

M. Gabriella Kuhn and Tim Steger

Let Γ be a free nonabelian group on $r \geq 2$ generators, Ω its boundary and $C(\Omega)$ the C^* algebra of complex valued continuos functions on Ω.

A unitary representation (π, H_π) of Γ is called tempered if it factors through $C^*_{\text{reg}}(\Gamma)$, the reduced C^*-algebra of Γ.

Any tempered representation π of Γ extends to a unitary representation $(\pi', \mathcal{H}')$ of the crossed product C^* algebra $\Gamma \ltimes_\lambda C(\Omega)$.

$(\pi'\mathcal{H}')$ is called a *boundary realization of* π; observe that in general the representation space H_π must be enlarged.

π' is called a **perfect** boundary realization of π if one can take $\mathcal{H}' = H_\pi$

In the last years (see [1], [2], [3]) we have produced a wide class of tempered representations of Γ, arising from irreducible matrix systems (V_a, H_{ba}, B_a), for which we can prove the following

Theorem 1. *Let π_M be constructed from an irreducible matrix system (V_a, H_{ba}, B_a). Then one and only one of the following possibilities holds*

1. *π_M admits* **only one perfect** *realization. In this case we say that π_M satisfies* **monotony**.

2. *π_M admits* **exactly two** *distinct perfect realizations (meaning that the same Γ representation has two extensions $\pi'_\pm$ which are equivalent as Γ representations but inequivalent as $\Gamma \ltimes_\lambda C(\Omega)$ representations). In this case we say that π_M satisfies* **duplicity**.

3. *π_M admits only* **one** *boundary realization which is* **not** *perfect. Moreover this realization is the direct sum of* **two** *inequivalent Γ representations. In this case we say that π_M satisfies* **oddity**.

As far as we know, the only representations in the literature that are not equivalent to some π_M (or some quotient of π_M for a suitable matrix system) are those coming from the restricion to Γ of the principal series of $SL(2,\mathbb{R})$. Nonetheless we can prove that our Theorem holds also for them and hence we conjecture that it is true for a generic tempered irreducible representation of Γ.

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Gradient estimates of harmonic functions and transition densities for Lévy processes

Tadeusz Kulczycki

We prove gradient estimates for harmonic functions with respect to a d-dimensional unimodal pure-jump Lévy process under some mild assumptions on the density of its Lévy measure. These assumptions allow for a construction of an unimodal Lévy process in $\mathbb{R}^{d+2}$ with the same characteristic exponent as the original process. The relationship between the two processes provides a fruitful source of gradient estimates of transition densities. We also construct another process called a difference process which is very useful in the analysis of differential properties of harmonic functions. The talk is based on the paper: T. Kulczycki, M. Ryznar "Gradient estimates of harmonic functions and transition densities for Lévy processes", to appear in Trans. Amer. Math. Soc.

Two-sided bounds for L_p-norms of combinations of products of independent random variables

Rafał Latała

We show that for every positive p, the L_p-norm of linear combinations (with scalar or vector coefficients) of products of i.i.d. nonnegative random variables with the p-norm one is comparable to the l_p-norm of the coefficients and the constants are explicit. As a result the same holds for linear combinations of Riesz products. We also present upper and lower bounds of the L_p-moments of partial sums of perpetuities. The talk is based on the joint work with Ewa Damek, Piotr Nayar and Tomasz Tkocz.

Sobolev spaces for Jacobi expansions

Bartosz Langowski

We define and study Sobolev spaces associated with Jacobi expansions. We prove that these Sobolev spaces are isomorphic, in the Banach space sense, with potential spaces (for the Jacobi 'Laplacian') of the same order. We also study mutual relations between various variants of Sobolev spaces depending on the choice of operators playing the role of higher order derivatives in their definitions.

Spcectral multipliers on 2-step groups: recent progress

Alessio Martini

For a homogeneous sublaplacian L on a stratified group G, a theorem due to Mauceri and Meda [6] and Christ [1] states that an operator of the form $F(L)$ is of weak type $(1,1)$ and bounded on $L^p(G)$ for $1 < p < \infty$ if the multiplier F satisfies a scale-invariant smoothness condition of order $s > Q/2$, where Q is the homogeneous dimension of G. For a long time it has been known that, in the case of a Heisenberg-type group G, the smoothness condition can be pushed down to $s > d/2$, where d is the topological dimension of G [4,10]. Analogous results have been obtained for more general operators than sublaplacians [5] or in other settings than stratified groups [2,3]. However it is still an open question whether the same improvement of the Christ–Mauceri–Meda theorem holds for a homogeneous sublaplacian on an arbitrary 2-step stratified group. I will report on recent results and new methods, developed in joint work with Detlef Müller [7,8,9]. In particular, we can prove that the condition $s > d/2$ is sufficient for all the $2-$step groups with $d \leq 7$ or with 2-dimensional second layer.

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On perpetuities arising in population genetics

Alexander Marynych

A random variable X is called perpetuity if its distribution $L(X)$ satisfies the fixed point equation $L(X) = L(MX + Q)$, where (M, Q) is a random vector which is assumed independent of X. Perpetuities is a classical object of pure and applied probability. They have received much attention due to the numerous applications in insurance, financial mathematics, ecology, biology, physics etc. In the talk I shall explain how perpetuities arise in the modern theory of population genetics. I shall focus on exchangeable coalescence and in particular on the structure of the coalescent trees. It will be shown that perpetuities come up as limiting objects (as the number of individuals in population tends to infinity) for some functionals on the coalescent tree.

Precise large deviation results for products of random matrices

Sebastian Mentemeier
joint work with Dariusz Buraczewski

Let (A_n) be a sequence of iid nonnegative matrices. The Furstenberg-Kesten theorem provides us with a strong law of large numbers, valid for all unit vectors x with nonnegative entries, namely

$$\lim_{n\to\infty} \frac{S_n^x}{n} := \lim_{n\to\infty} \frac{1}{n} \log \|A_n \dots A_1 x\| = \gamma \qquad \mathbb{P}\text{-f.s.},$$

with γ being the top Lyapunov exponent of the sequence (A_n). Later on, Hennion proved a CLT for the sequence $(S_n^x - n\gamma)$. In both cases, the asymptotics are independent of the particular choice of x.

We prove a large deviation result similar to the Bahadur-Rao theorem, i.e. for (suitable) $q > 0$, there is explicitly given a sequence $J_n(q)$ tending to infinity at an exponential rate, such that

$$\lim_{n\to\infty} J_n(q) \mathbb{P}\left(\frac{S_n^x}{n} - \gamma \geq q\right) = r(x)$$

for a positive continuous function r. Besides giving a sharp rate of convergence for the LLN, our result describes for the first time, how fluctuations depend on the starting vector x.

This result can be applied to identify the precise tail behavior of stationary solutions of stochastic recursions, such as random difference equations or the smoothing transform.

S-inequality for a certain product measures

Piotr Nayar

Let us consider a Borel product probability measure μ on the Euclidean space R^n and a given family of subsets K. Our goal is to find the minimum value of the quantity $\mu(tA), t \geq 1$ among all sets A in K with fixed measure. In this talk we summarize old and recent results concerning this problem. We also indicate some applications in probability theory, e.g., to the so-called Khinchin-Kahane-type inequalities.

Potential operators associated with various orthogonal expansions.

Adam Nowak

We overview recent results concerning counterparts of the classical Riesz and Bessel potentials in the settings of Jacobi, Hermite, Laguerre, Dunkl-Laguerre, discrete and continuous Fourier-Bessel, and Dunkl-Fourier-Bessel expansions. The results, obtained in collaboration with K. Stempak and L. Roncal, embrace sharp estimates for the potential kernels and complete descriptions of $L^p - L^q$ mapping properties of the potential operators. The latter may be regarded as sharp analogues of the celebrated Hardy-Littlewood-Sobolev theorem in the above mentioned contexts.

Some sharp restriction inequalities

Diogo Oliveira e Silva

The goal of this talk will be to find the sharp forms and characterize the complex-valued extremizers of the adjoint Fourier restriction inequalities on the sphere:

$$\left\|\widehat{f\sigma}\right\|_{L^p(\mathbb{R}^d)} \lesssim \|f\|_{L^q(\mathbb{S}^{d-1},\sigma)},$$

in the cases $(d,p,q) = (d,2k,q)$ with $d,k \in \mathbb{N}$ and $q \in \mathbb{R}^+ \cup \{\infty\}$ satisfying:

(a) $k = 2$, $q \geq 2$ and $3 \leq d \leq 7$;

(b) $k = 2$, $q \geq 4$ and $d \geq 8$;

(c) $k \geq 3$, $q \geq 2k$ and $d \geq 2$.

Tools include a spherical harmonic decomposition, the study of the Cauchy-Pexider functional equation for sumsets of the sphere, and a sharp multilinear weighted restriction inequality with weight related to the k-fold convolution of the surface measure.

This builds up on recent results by D. Foschi, and is joint work with E. Carneiro.

On the characterization of Gelfand-Shilov-Roumieu spaces

Mihai Pascu

Generalized $\mathbf{m}$-Gelfand-Shilov-Roumieu (GSR) vector spaces $\mathcal{S}_{\mathbf{m}}(\mathbf{X})$ are introduced. Here $\mathbf{m} = (m^{(1)}, ..., m^{(n)})$, $\mathbf{X} = (X_1, ..., X_n)$ and $m^{(1)}, ..., m^{(n)}$ are sequences of positive numbers and $X_1, ..., X_n$ are operators in a Hilbert space. Our definition extends ter Elst's definition of Gevrey vector spaces [1]. Conditions are given on the sequences $m^{(1)}, ..., m^{(n)}$ and on the operators $X_1, ..., X_n$ so that the equality $S_{\mathbf{m}}(\mathbf{X}) = S_{m^{(1)}}(X_1) \cap ... \cap S_{m^{(n)}}(X_n)$ is valid. As a corollary we obtain a proof of a characterization theorem for GSR spaces.

Gelfand-Shilov-Roumieu spaces can also be defined by using, instead sequences of positive numbers, their associated functions. We provide a characterization of those weight functions which may be used for the definition of GSR spaces.

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Poisson kernels on nilpotent, 3-meta-abelian groups

Richard Penney and Roman Urban

A Lie Group N is 3-meta-abelian if

$$N = N_1 \rtimes (N_2 \rtimes N_3)$$

where the N_j are abelian. Let S be a semi direct product $S = N \rtimes A$ where N is a connected and simply connected, nilpotent, 3-abelian Lie group and A is isomorphic with R_k, $k > 1$. We assume that the A-action on each N_j is diagonal. On S

we consider a class of second order left-invariant differential operators of the form $\mathcal{L}_\alpha = L_a + \Delta_\alpha$, where $\alpha \in \mathbb{R}^k$, and for each $a \in R^k$, L^a is left-invariant second order differential operator on N and $\Delta_\alpha = \Delta - \langle \alpha, \nabla \rangle$, where Δ is the usual Laplacian on $\mathbb{R}^k$. We make use of a probabilistic description of the heat semi-group on S to prove a strong upper bound for the Poisson kernel for the operator $\mathcal{L}_\alpha$.

Decay of coefficients of unitary representations of negatively curved groups

Christophe Pittet

The goal of the talk is to explain how certain tools from dynamics (Patterson-Sullivan measures) and from geometry (Busemann functions and Gromov products) provide estimates (due to U. Bader and R. Muchnik) of the elementary spherical function of a CAT(-1) space, generalizing the classical case of rank 1 symmetric spaces.

Topology on the unitary dual of exponential Lie groups – the case of complete solvability

Detlev Poguntke

For a so-called exponential Lie group G (with Lie algebra $\mathfrak{g}$) the unitary dual $\hat{G}$ can be determined by an extension of Kirillov's orbit method (originally formulated for nilpotent Lie groups). The dual $\hat{G}$ is in bijective correspondence with the orbit space $\mathfrak{g}^*/G$, where $\mathfrak{g}^*$ denotes the linear dual of, $\mathfrak{g}$, and G acts on $\mathfrak{g}^*$ via the coadjoint representation. It is natural to expect that this correspondence is a homeomorphism. This was shown to be true in a monography (in 1994) by H. Leptin and J. Ludwig. At present, I am reconsidering their proof. The case of completely solvable Lie groups is finished, and I shall report on it. The general case is "work in progress".

L^p-integrability of Fourier transform of distributions supported on thin sets

Senthil Raani K.S.

In this talk we relate the support of the tempered distribution and L^p-integrability of its Fourier transform. It is proved that there does not exist any non zero function in $L^p(\mathbb{R}^n)$ with $p \leq 2n/\alpha$ if its Fourier transform is supported by a set of finite packing α-measure. It is shown that the assertion fails for $p > 2n/\alpha$ where $0 < \alpha < n$. The result is applied to prove L^p Wiener-Tauberian theorems for $\mathbb{R}^n$ and $M(2)$.

Limit transition for hypergeometric functions associated with root systems

Margit Rösler

Hypergeometric functions associated with root systems generalize the spherical functions of Riemannian symmetric spaces as well as classical special functions of hypergeometric type like Jacobi functions. In this talk, we shall present limit transitions from hypergeometric functions of type BC to such of type A, as the multiplicity parameters tend to infinity in a suitable way. For particular multiplicity values, BC-type hypergeometric functions occur as spherical functions of Grassmann manifolds, and our limits can be interpreted in the context of infinite-dimensional symmetric spaces.

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The Dirichlet problem on hyperbolic space

Grzegorz Serafin

Let us consider the hyperbolic Brownian motion with drift on the hyperbolic space $\mathbb{H}^n = \{x \in \mathbb{R}^n : x_n > 0\}$. The generator of the process is $\frac{1}{2}\Delta_\mu$, where

$$\Delta_\mu = x_n^2 \sum_{k=1}^{n} \frac{\partial^2}{\partial x_k^2} - (2\mu - 1)x_n \frac{\partial}{\partial x_n}, \qquad \mu > 0.$$

Note that $\Delta_{\frac{n-1}{2}}$ is the Laplace-Beltrami operator on $\mathbb{H}^n$.

We discuss the λ-Poisson kernel and the λ-Green function ($\lambda \geq 0$) of a Lipschitz domain $U \subset \mathbb{H}^n$, which play an essential role in the context of the Dirichlet problem for the set U and the operator $\frac{1}{2}\Delta_\mu - \lambda$. For bounded sets we show how the case $\lambda \geq 0$ can be reduced to $\lambda = 0$. Then, we reformulate and solve the Dirichlet problem for unbounded sets. As an example we consider the strip $S_a = \{x \in \mathbb{H}^n : 0 < x_1 < a\}$, $a > 0$, and give uniform estimates of the aforementioned objects.

Roe-Strichartz Theorem on a class of two-step nilpotent Lie groups

Suparna Sen

Generalizing a result of J. Roe for functions on the real line, Strichartz proved that if a doubly infinite sequence of uniformly bounded functions $\{f_k\}_{k\in\mathbb{Z}}$ on $\mathbb{R}^n$ satisfy $f_{k+1} = \Delta f_k$ for each k where Δ is the Laplacian on $\mathbb{R}^n$, then $\Delta f_0 = -f_0$. He also proved a similar result on Heisenberg group. We give a generalization of this result to a class of two-step nilpotent Lie groups.

Gaussian estimate for inhomogeneous random walk on the positive quadrant

Mohamed Sifi
with N. Ben Salem and S. Mustapha

We obtain an upper Gaussian estimate for the transition kernel corresponding to a inhomogeneous random walk on the positive quad- rant. Our results hold under some strong but natural assumptions of symmetry of the increments and ellipticity. To this end, we develop discrete variant of the boundary Harnack principle based on potential- theoretical tools.

On one generalization of the martingale transform and its applications to paraproducts and stochastic integrals

Kristina Ana Škreb

In this talk we introduce a variant of Burkholder's martingale transform associated with two martingales with respect to two different, yet compatible filtrations. Even though the classical martingale techniques cannot be applied, the discussed transformation still satisfies some expected L^p estimates. These estimates are established by a variant of the Bellman function technique, as it was recently developed in the theory of multilinear singular integrals.

Then we apply the obtained inequalities to general-dilation twisted paraproducts, particular instances of which have already appeared in the literature. As another application, we construct stochastic integrals $\int_0^t H_s d(X_s Y_s)$ associated with certain martingales $(X_t)_{t\geq 0}$ and $(Y_t)_{t\geq 0}$. Therefore, the process $(X_t Y_t)_{t\geq 0}$ is shown to be a "good integrator", although it is not necessarily a semimartingale, or even adapted to any convenient filtration. We emphasize that the key step in the construction is to replace the Itō isometry with our bilinear estimate for discrete martingales.

This is a joint work with Vjekoslav Kovač (University of Zagreb).

Littlewood–Paley Theory for $\tilde{A}_2$-buildings

Tim Steger

The most common version of Littlewood–Paley theory on the plane $\mathbf{R}^2$ depends on the usual 1-parameter family of dilations. Looking at the plane as a product, $\mathbf{R} \times \mathbf{R}$, one has another Littlewood–Paley theory, depending on the 2-parameter family of dilations. The Heisenberg group also admits a 2-parameter family of dilations, and the work of [Steger–Trojan] looks at Littlewood–Paley theory adapted to that family. We work in the p-adic context, where there is one particularly nice choice for the maximal (resp. square) function. The "rectangles" used to define the maximal (resp. square) function are parameterized by the vertices of an $\tilde{A}_2$ building, so if you believe that p-adic numbers really exist, you are forced to believe in buildings too.

Spectral properties of a class of unbounded Jacobi matrices

Grzegorz Świderski

Spectral properties of Jacobi matrices are closely related to those of the measure of orthogonality of the associated orthogonal polynomials. After explaining this connection, we concentrate on special class of polynomials with regular behaviour of coefficients appearing in their recurrence relation. We present Chihara's conjecture about the support of the measure of orthogonality. Next we show generalizations of Dombrowski' theorems and show applications to Chihara's conjecture.

Harmonic analysis operators in certain Dunkl setting

Tomasz Z. Szarek

In the talk we will consider several harmonic analysis operators in the multi-dimensional context of the Dunkl Laplacian (the case when a group of reflections is isomorphic to $\mathbb{Z}_2^n$). Our investigations include maximal operators, multipliers of Laplace and Laplace-Stieltjes transform type, Riesz transforms, g-functions and Lusin area integrals. Using the general Calderón-Zygmund theory we prove that these objects are bounded in weighted L^p spaces, $1 < p < \infty$, and are of weighted weak type $(1, 1)$. In our main result we do not assume positivity of the multiplicity function, which is unusual in the Dunkl context.

The talk is based on the joint paper with A.J. Castro [1].

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Perturbations of Lévy Processes

Karol Szczypkowski

A perturbation series is an explicit method of constructing new semigroups or fundamental solutions. It is thus of the interest to obtain its upper and lower bounds.

I will present recent results on 3P and 4G inequalities for transition densities and discuss its consequences to perturbation theory. In particular, the extensions of theorems of [4], [5], where the perturbation series is estimated, are available.

The talk is based on [2], [1] and [3].

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Limit theorems on nilpotent groups, global law of large numbers and large deviations

Ryokichi Tanaka

We show the global law of large numbers for random walks with finite first moments on nilpotent Lie groups, and large deviations results for lattice walks in connection with the Carnot-Caratheodory metric.

On Hermite pseudo-multipliers

Sundaram Thangavelu

In this talk we plan to discuss some recent results on Hermite psudo-multipliers obtained in collaboration with my student Sayan Bagchi. Unlike the case of multipliers, there is not much known about Hermite psudo-multipiers which in some sense are analogous to pseudo-differential operators. We address the problem of finding sufficient conditions on the symbol so that the pseudo-multiplier is bounded on $L^p(\mathbb{R}^n)$.

The Hilbert transform along vector fields

Christophe Thiele

An old conjecture by A. Zygmund proposes a Lebesgue Differentiation theorem along a Lipschitz vector field in the plane. E. Stein formulated a corresponding conjecture about the Hilbert transform along the vector field. If the vector field is constant along vertical lines, the Hilbert transform along the vector field is closely related to Carleson's operator. We discuss some progress in the area by and with Michael Bateman and by my student Shaoming Guo.

Discrete maximal functions in higher dimensions

Bartosz Trojan

We present higher dimensional counterpart of Bourgain's pointwise ergodic theorem along an arbitrary integer valued polynomial mapping $\mathcal{P} : \mathbb{Z}^k \to \mathbb{Z}^d$. We achieve this by proving variational estimates V_r on $\ell^p\Big(\mathbb{Z}^d\Big)$ for averaging operator

$$M_N f(x) = N^{-k} \sum_{|y_1| \le N} \cdots \sum_{|y_k| \le N} f\Big(x - \mathcal{P}(y)\Big).$$

This work is joint with Mariusz Mirek (Uniwersytet Wrocławski).

Central limit theorems for random walks associated with Grassmann manifolds

Michael Voit

By the theory of Heckman-Opdam, the spherical functions on the non-compact Grassmann manifolds $G_{p,q}(\mathbb{F})$ with $p \ge q$ and rank q over the fields $\mathbb{F} = \mathbb{R}, \mathbb{C}, \mathbb{H}$ can be described as hypergeometric functions of type BC on the Weyl chamber

$$C_q := \{x \in \mathbb{R}^q : \, x_1 \ge x_2 \ge \cdots \ge x_q \ge 0\}.$$

Moreover, the compact Grassmann manifolds lead to Jacobi polynomials of type BC on the associated alcoves. In both cases, the associated product formulas and Harish-Chandra integral representations can be written down explicitly for integers $p \ge 2q - 1$ such that they can be extended analytically to all real parameters $p \ge$

$2q-1$. These integral representations lead to several sharp limit transitions for the spherical functions and finally to central limit theorems.

In the talk we focus on a Hilb-type formula which relates the Jacobi polynomials with Bessel functions on C_q. We show that this asymptotic result leads to a central limit theorem for random walks on the dual spaces of the compact Grassmannians which can be identified with the discrete Wely chamber

$$P^+ := \{\lambda \in (2\mathbb{Z}^+)^q : \quad \lambda \in C_q\}.$$

The limits will be distributions of corresponding Laguerre ensembles on C_q which are well-known from random matrix theory. For $q = 1$ this CLT is known for a long time.

Besides this CLT we sketch further applications to CLTs for random walks on the non-compact Grassmann manifolds and on C_q. All results are joint work with M. Rösler.

A variant on affine-invariant harmonic analysis

Jim Wright

Here we begin with a survey of an interesting area known as affine-invariant harmonic analysis and end with an 'automorphic-invariant' result on the Heisenberg group.

Dimension free L^p estimates for Riesz transforms via an H^∞ joint functional calculus

Błażej Wróbel

In 1983 E. M. Stein proved dimension free L^p bounds for classical Riesz transforms on $\mathbb{R}^d$. Since then many authors studied the phenomenon of dimension free estimates for Riesz transforms defined in various contexts. In this talk we present a fairly general scheme for deducing the dimension free L^p boundedness of d-dimensional Riesz transforms from the L^p boundedness of one-dimensional Riesz transforms. The crucial tool we use is an H^∞ joint functional calculus for strongly commuting operators. The scheme is applicable to all Riesz transforms acting on 'product' spaces, e.g.: Riesz transforms connected with (classical) multi-dimensional orthogonal expansions and discrete Riesz transforms on products of groups having polynomial growth.

A Fourier Analysis of Extremal Events

Yuwei Zhao

The extremogram is an asymptotic correlogram for extreme events constructed from a regularly varying strictly stationary sequence. Correspondingly, the spectral density generated from the extremogram is introduced as a frequency domain analog of the extremogram. Its empirical estimator is the extremal periodogram. The extremal periodogram shares numerous asymptotic properties with the periodogram of a linear process in classical time series analysis: the asymptotic distribution of the periodogram ordinates at the Fourier frequencies have a similar form and smoothed versions of the periodogram are consistent estimators of the spectral density. By proving a functional central limit theorem, the integrated extremal periodogram can be used for constructing asymptotic tests for the hypothesis that the data come from a strictly stationary sequence with a given extremogram or extremal spectral density. A numerical method, the stationary bootstrap, can be applied to the estimation of the integrated extremal periodogram.