Perspectives of Modern Complex Analysis

July 21–25, 2014, Będlewo, Poland

Abstracts of talks

Tomasz Adamowicz (Institute of Mathematics of the Polish Academy of Sciences)

Elliptic systems of PDEs in the plane

We will discuss various geometric properties of p-harmonic functions and mappings in the plane and beyond it. Topics of the presentation include level sets of component functions of a p-harmonic mapping, convexity properties and relations between elliptic PDEs and quasiregular mappings.

Abhijit Banerjee (University of Kalyani)

On the unique and bi-unique range sets for meromorphic functions

Let f and g be two non-constant meromorphic functions and let a be a finite complex number.

We say that f and g share a CM, provided that $f-a$ and $g-a$ have the same zeros with the same multiplicities. Similarly, we say that f and g share a IM, provided that $f-a$ and $g-a$ have the same zeros ignoring multiplicities. In addition we say that f and g share ∞ CM, if $1/f$ and $1/g$ share 0 CM, and we say that f and g share ∞ IM, if $1/f$ and $1/g$ share 0 IM.

Let S be a set of distinct elements of $\mathbb{C}\cup\{\infty\}$ and $E_f(S)=\bigcup_{a\in S}\{z: f(z)-a=0\}$, where each zero is counted according to its multiplicity. If we do not count the multiplicity the set $E_f(S)=\{z: f(z)-a=0\}$ is denoted by $\overline{E}_f(S)$. If $E_f(S)=E_g(S)$ we say that f and g share the set S CM. On the other hand if $\overline{E}_f(S)=\overline{E}_g(S)$, we say that f and g share the set S IM. Evidently, if S contains only one element, then it coincides with the usual definition of CM (respectively, IM) shared values.

Let a set $S\subset\mathbb{C}$ and f and g be two non-constant meromorphic (entire) functions. If $E_f(S)=E_g(S)$ implies $f\equiv g$ then S is called a unique range set for meromorphic (entire) functions or in brief URSM (URSE). This notion was introduced by F. Gross and C. C. Yang in [1].

In the last couple of years the concept of URSE, URSM have caused an increasing interest among the researchers. The study is focused mainly on two problems: finding different Unique Range Sets (URS) with minimum cardinality and characterizing an URS. The advent of the gradation of sharing of values and sets known as weighted sharing by I. Lahiri [2] further expedite the research in this regard. Li and Yang [3] first pointed out the fact that the finite URSM's are the set of distinct zeros of some suitable polynomials and so one can not deny the importance of the underlying polynomial. This realization has recently motivated the researchers to investigate more judiciously the characteristics of the polynomial backbone of a finite URSM (URSE).

In 2003, the following question was asked by Lin and Yi [3].

Question A. *Can one find two finite sets S_j $(j=1,2)$ such that any two non-constant meromorphic functions f and g satisfying $E_f(S_j)=E_g(S_j)$ for $j=1,2$ must be identical ?*

The above question motivates the present author to introduce the analogous definition of URS namely Bi-Unique Range Sets (BURS) as follows: A pair of finite sets S1 and S2 in C is called bi unique range sets for meromorphic (entire) functions or in brief BURSM (BURSE) if for any two non-constant meromorphic (entire) functions f and g, $E_f(S_1) = E_g(S_1)$, $E_f(S_2) = E_g(S_2)$ implies $f \equiv g$. In this talk we propose to highlight different URS and BURS obtained so far and the scope for future investigations.

[1] F. Gross and C. C. Yang, *On pre-image and range sets of meromorphic functions*, Proc. Japan Acad. 58 (1982), 17–20.

[2] I. Lahiri, *Weighted sharing and uniqueness of meromorphic functions*, Nagoya Math. J. 161 (2001), 193–206.

[3] P. Li and C. C. Yang, *Some further results on the unique range sets of meromorphic functions*, Kodai Math. J. 18 (1995), 437–450.

[4] W. C. Lin and H. X. Yi, *Some further results on meromorphic functions that share two sets*, Kyungpook Math. J. 43 (2003), 73–85.

Carl Bender (Washington University in St. Louis)

Nonlinear eigenvalue problems and PT-symmetric quantum mechanics

We discuss new kinds of nonlinear eigenvalue problems that are associated with instabilities, separatrix behavior, and hyperasymptotics. We consider first the toy differential equation $y'(x) = \cos[\pi x y(x)]$, which arises in a number of physical contexts. We show that the initial condition $y(0)$ falls into discrete classes: $a_{n-1} < y(0) < a_n$ $(n = 1, 2, 3, \ldots)$. If $y(0)$ is in the nth class, $y(x)$ exhibits n oscillations. The boundaries a_n of these classes are strongly analogous to quantum-mechanical eigenvalues and finding the large-n behavior of a_n is analogous to performing a high-energy semiclassical (WKB) approximation in quantum mechanics. For large n, a_n is asymptotic to $A\sqrt{n}$, where $A = 2^{5/6}$. Surprisingly, the constant A is numerically close to the lower bound on the power-series constant P, which plays a fundamental role in the theory of complex variables and which is associated with the asymptotic behavior of zeros of partial sums of Taylor series.

The first two Painlevé transcendents P1 and P2 have instabilities, separatrices, and eigenvalue structures like those of $y'(x) = \cos[\pi x y(x)]$. As $n \to \infty$, the nth eigenvalue for P1 grows like $Bn^{3/5}$ and the nth eigenvalue for P2 grows like $Cn^{2/3}$. We calculate the constants B and C analytically by reducing the Painlevé transcendents to linear eigenvalue problems already studied and well known in PT-symmetric quantum theory.

Michael Benedicks (KTH Royal Institute of Technology)

Parameter selection in one-dimensional maps revisited

There are several ways to co construct abundance (positive measure) of parameters with absolutely continuous invariant measures for one-dimensional maps. The first is the famous construction due to Jakobson. Another is due to Carleson and myself in the case of the quadratic family. I will discuss this later proof in view of its basic ingredients, and which generalizations may be possible in the one-dimensional real and complex case.

Walter Bergweiler (Christian-Albrechts-Universität zu Kiel)

Entire functions in the Eremenko–Lyubich class which have bounded Fatou components

(joint work with Núria Fagella and Lasse Rempe-Gillen)

The Eremenko–Lyubich class B consists of all transcendental entire functions f for which the set $\text{sing}(f^{-1})$ of critical and (finite) asymptotic values is bounded. A function $f \in B$ is called hyperbolic if every point of the closure of $\text{sing}(f^{-1})$ is contained in an attracting periodic basin. We show that if a hyperbolic map $f \in B$ has no asymptotic value and every Fatou component of f contains at most finitely many critical points, then every Fatou component of f is bounded. Moreover, the Fatou components are quasidisks in this case. If, in addition, there exists N such that every Fatou component contains at most N critical points, then the Julia set of f is locally connected.

For hyperbolic maps in B with only two critical values and no asymptotic value we find that either all Fatou components are unbounded, or all Fatou components are bounded quasidisks.

We illustrate the results by a number of examples. In particular, we show that there exists a hyperbolic entire function $f \in B$ with only two critical values and no asymptotic value for which all Fatou components are bounded quasidisks, but the Julia set is not locally connected.

Lev Birbrair (Universidade Federal do Ceara)

Mumford theorem in Lipschitz geometry

(joint work with Le Dung Trang, Alexandre Fernandes and Edson Sampaio)

The classical Theorem of Mumford states that a topologically regular Complex Algebraic Surface in $\mathbb{C}^3$ with an isolated singular point is smooth. We proof that any Lipschitz Regular Complex Algebraic set is smooth. No restriction on the dimension is needed. No restriction of Singularity to be isolated is needed.

Christoph Böhm (Julius-Maximilians-Universität Würzburg)

A Komatu–Loewner equation for mutliple slits

We give a generalization of Komatu–Loewner equations to multiple slits. Therefore we will consider the radial case, i.e. we take circular slit disks as our standard domains. Komatu–Loewner equations are differential equations for certain normalized conformal mappings that can be used to describe the growth of slits within multiply connected domains. We show that it is possible to choose constant coefficients in this equations in order to generate given disjoint slits and that those coefficients are uniquely determined under a suitable normalization of the differential equation.

Mario Bonk (University of California, Los Angeles)

Quasisymmetric uniformization and rigidity

Uniformization by quasisymmetric maps and rigidity phenomena for such maps are linked to questions in geometric group theory and and in complex dynamics. In my talk I will give a survey on some recent developments.

Carlos Cabrera (Universidad Nacional Autónoma de México)

On Poincaré extensions of rational maps

(joint work with Peter Makienko and Guillermo Sienra)

We will discuss some examples of extensions of rational maps to the hyperbolic 3-space. In particular, in the Fuchsian case, for Blaschke products, we give extensions that map the semigroup of Blaschke maps homorphically into the semigroup of endomorphisms of the hyperbolic 3-space.

Igor Chyzhykov (Cardinal Stefan Wyszyński University in Warsaw)

On the growth, zero distribution and factorization of analytic functions of moderate growth in the unit disc

Let f be an analytic function in $\mathbb{D} = \{z : |z| < 1\}$, $\rho_M[f]$ and $\rho_T[f]$ be the orders of f defined by the logarithm of maximum modulus and the Nevanlinna characteristic, respectively. We then define for $p \geq 1$

$$m_p(r,f) = \left(\frac{1}{2\pi} \int_0^{2\pi} |\log|f(re^{i\theta})||^p \, d\theta \right)^{\frac{1}{p}}, \quad 0 < r < 1.$$

We write (see [2])

$$\rho_p[f] = \limsup_{r \uparrow 1} \frac{\log m_p(r,f)}{-\log(1-r)}.$$

Note that $\rho_1[f] = \rho_T[f]$, and ([1]) $\rho_M[f] \leq \rho_p[f] + \frac{1}{p}$ $(p \geq 1)$.

We define ρ_∞-*order* of f as

$$\rho_\infty[f] = \lim_{p \to \infty} \rho_p[f].$$

It is clear that, $\rho_M[f] \leq \rho_\infty[f]$. Moreover, Linden [1] proved that $\rho_\infty[f] = \rho_M[f]$ provided that $\rho_M[f] \geq 1$.

The talk is devoted to applications of the concept of ρ_∞-order to studying zero distribution, factorization, minimum modulus of analytic functions in the unit disc, especially of order $\rho_M[f] \leq 1$ (see [3]–[5]).

[1] C. N. Linden, *Integral logarithmic means for regular functions*, Pacific J. of Math. 138 (1989), no. 1, 119–127.

[2] C. N. Linden, *The characterization of orders for regular functions,* Math. Proc. Cambridge Phil. Soc. 111 (1992), no.2, 299–307.

[3] I. Chyzhykov, J. Heittokangas, J. Rättyä, *Sharp logarithmic derivative estimates with applications to ODE's in the unit disc,* J. Austr. Math. Soc. 88 (2010), 145–167.

[4] Igor Chyzhykov, Severyn Skaskiv, *Growth, zero distribution and factorization of analytic functions of moderate growth in the unit disc,* in the Blaschke Products and their Applications. J. Mashreghi, E. Fricain eds., Fields Institute Communications, V.65. (2012), 159–173.

[5] I. E. Chyzhykov, *Zero distribution and factorization of analytic functions of slow growth in the unit disc*, Proc. Amer. Math. Soc. 141 (2013), 1297–1311.

Ewa Ciechanowicz (University of Szczecin)

Asymptotic values and asymptotic functions of entire and meromorphic functions

We call a value $a \in \overline{\mathbb{C}}$ *an asymptotic value* of a meromorphic function f if there exists a continuous curve $\Gamma \subset \mathbb{C}$, $\Gamma : z = z(t), 0 \leq t < \infty, z(t) \to \infty$ for $t \to \infty$, such that $\lim_{z \to \infty, z \in \Gamma} (f(z) - a) = \lim_{t \to \infty} f(z(t) - a) = 0$. A pair $\{a, \Gamma\}$, defined as above, is called *an asymptotic spot of* f. A classical theorem of Denjoy-Carleman-Ahlfors gives a sharp upper estimate of the number of asymptotic spots in terms of order.

Theorem A. *An entire function of finite lower order λ cannot have more than $[2\lambda]$ different finite asymptotic values or even $[2\lambda]$ different asymptotic spots with finite asymptotic values.*

For entire functions of infinite order the set of asymptotic values may be infinite, even equal with $\overline{\mathbb{C}}$ (an example was given by Gross in 1918). The set of asymptotic values of a meromorphic function even of finite order may be infinite (an example was given by Eremenko in 1986). Denjoy also set up the following question:

If an entire function f has got k distinct asymptotic functions of order less that $1/2$ is the order $\varrho(f) \geq k/2$?

The problem is still open. The best result in this direction was obtained by Fenton in 1983.

In 2004 Marchenko introduced the following definition. We say that $a \in \overline{\mathbb{C}}$ is *an α_0-strong asymptotic value* of a meromorphic function f if there exists a continuous curve $\Gamma: \ z = z(t),\ 0 \leq t < \infty,\ z(t) \to \infty$ as $t \to \infty$, such, that

$$\liminf_{t\to\infty} \frac{\log|f(z(t)) - a|^{-1}}{T(|z(t)|, f)} = \alpha(a) \geq \alpha_0 > 0, \qquad \text{if} \quad a \neq \infty;$$

$$\liminf_{t\to\infty} \frac{\log|f(z(t))|}{T(|z(t)|, f)} \geq \alpha_0 > 0, \qquad \text{if} \quad a = \infty.$$

If a is an α_0-strong asymptotic value of f, then an asymptotic spot $\{a, \Gamma\}$ is called *an α_0-strong asymptotic spot.* At the same time Marchenko presented the following estimate.

Theorem B [1]. *Let f be a meromorphic function of finite lower order λ and $\{a_\nu, \Gamma_\nu\}$, $\nu = 1, 2, ..., k$, - distinct α_0-strong asymptotic spots of f. Then $k \leq \left[\frac{2B(\lambda)}{\alpha_0}\right]$, where $B(\lambda)$ is Paley's constant.*

Similar estimates are possible for strong asymptotic functions.

Theorem 1 [2]. *Let f be a transcendental meromorphic function of finite lower order λ, such that $N(r, f) = o(T(r, f))$ for $r \to \infty$, possibly outside an exceptional set of finite linear measure. The number m of distinct α_0-strong rational asymptotic spots of f is finite and $m \leq \left[\frac{B(\lambda)}{\alpha_0}\right]$.*

Theorem 2 [2]. *Let f be a meromorphic function of finite lower order λ. For polynomials of a degree not overcoming d, the number m of distinct α_0-strong polynomial asymptotic spots of f is finite and $m \leq \left[\frac{(d+2)B(\lambda)}{\alpha_0}\right]$.*

[1] I. I. Marchenko, *On the strong asymptotic spots of meromorphic functions of finite lower order*, Mat. fizika, analiz, geometriya 11 (2004), 484–491 (Russian).

[2] E. Ciechanowicz, I. I. Marchenko, *On deviations and strong asymptotic functions of meromorphic functions of finite lower order*, Journal of Math. Anal. and Appl. 382 (2011), 383–398.

Arthur Danielyan (University of South Florida)

Approximation by sequences of uniformly bounded polynomials

Let E be an arbitrary subset of the unit circle T in the complex plane and let f be a function defined on E. When there exist polynomials P_n which are uniformly bounded by a number $M > 0$ on T and converge (pointwise) to f at each point of E? We give a necessary and sufficient description of such functions, and also discuss some related questions.

Andrea del Monaco (Università degli Studi di Roma Tor Vergata)

Preimages of slits in $\mathbb{C}^n$

In dimension $n = 1$, it is well known that preimages of slits under univalent functions defined in the unit disk $\mathbb{D}$ are still slits. Unfortunately, this is not always the case in dimension $n \geq 2$. In this talk, we will give an example of such functions.

Neil Dobbs (University of Helsinki)

Pointwise Lyapunov exponents in one-dimensional dynamics

Given say a quadratic map of the interval, if is has an absolutely continuous invariant probability measure then the Lyapunov exponent of almost every point exists and is equal to the Lyapunov exponent of the measure, a positive constant. Conversely, if the upper (pointwise) Lyapunov exponent is positive almost everywhere, then there is an absolutely continuous invariant probability measure, and so the Lyapunov exponent exists almost everywhere. Related results hold for complex polynomials. We study this question for maps from the (complex) exponential family. For Misiurewicz parameters, pointwise exponents almost never exist, despite the existence of a conservative, ergodic absolutely continuous invariant measure.

Dzmitry Dudko (Georg-August-Universität Göttingen)

Expanding Thurston maps and matings

Christophe Dupont (Université de Rennes)

Rigidity of Kummer and Lattès examples

(joint work with Serge Cantat)

The talk concerns the ergodic theory of holomorphic automorphisms f of complex surfaces X. Our main result characterizes the pairs (X, f) whose measure of maximal entropy is absolutely continuous with respect to Lebesgue measure: such a system must be a Kummer example, i.e. a rational factor of an invertible linear mapping on a complex torus. A similar result was previously known for endomorphisms of complex projective spaces $\mathbb{CP}(k)$ involving Lattès maps (Zdunik, Mayer for $k = 1$, Berteloot, Loeb, Dupont for $k > 1$). As in these works, we use a technique of renormalization. Further arguments are however needed in the invertible case, using local product structure, compactness of entire curves and invariant foliations.

Adam Epstein (University of Warwick)

Deformation spaces of finite type maps

The seminal work of Eremenko–Lyubich showed how any finite type entire map belongs to a natural deformation space whose finite dimensionality is essential to the adaptation of both Sullivan's No Wandering Domains Theorem and the Fatou-Shishikura Inequalty. We discuss how related transversality results allow for a further extension to all finite type complex analytic maps.

Abdolmajid Fattahi (Razi University)

Banach frame in Banach space and stability

In 1991 Gröchernig [3] generalized Hilbert frames to Banach spaces and called them atomic decompositions. In this talk I consider Banach frame theory and I present a new definition for Banach frame. We use this new definition to investigate the properties of Hilbert frame which can be derived within the notion of Banach frame.

[1] O. Christensen, *Introduction to frames and Riesz bases*, Boston, Birkhauser 2003.

[2] R. J. Duffin and A. C. Schaeffer, *A class of nonharmonic Fourier series*, Trans. Amer. Math. Soc. 72 (1952), 341–366.

[3] K. H. Gröchenig, *Describing functions: frames versus atomic decompositions*, Monatshefte für Mathematik 112 (1991), 1–41.

Sergii Favorov (Karazin National University)

Blaschke-type condition in the unite disc and in unbounded domains, and their application in operator theory

(joint work with L. Golinskii)

It is known that zeros z_n of any bounded analytic function in the unit disc satisfy Blaschke condition $\sum(1-|z_n|) < \infty$. There are a lot of its generalization to unbounded analytic functions (M. M. Djrbashian, W. Hayman and B. Korenblum, F. A. Shamoyan, and many others).

We consider a new case of analytic functions f in the unit disc growing near a subset E of the boundary and obtain an analog of the above condition

$$\sum(1-|z_n|)\operatorname{dist}^{\kappa}(z,E) < \infty,$$

where $\kappa \geq 0$ depends on growth of f and Minkowski dimension of E. Our result is sharp.

Next, we introduce a notion of r-convexity for subsets of the complex plane. It is a pure geometric characteristic that generalizes the usual notion of convexity. We investigate analytic and subharmonic functions that grow near the boundary in unbounded domains with r-convex compact complement. We obtain the Blaschke-type bounds for its Riesz measure and, in particular, for zeros of unbounded analytic functions in such domains.

All our results are based on new estimates for the Green function.

Also, we apply our results to perturbation theory of linear operators in a Hilbert space. Namely, we find quantitative estimates for the rate of condensation of the discrete spectrum of a perturbed operator near its essential spectrum.

Alexandre Fernandes (Universidade Federal do Ceara)

Globally subanalytic constant mean curvature surfaces in $\mathbb{R}^3$

(joint work with J. L. Barbosa, L. Birbrair and M. P. do Carmo)

We prove that globally subanalytic nonsingular Constant Mean Curvature surfaces of $\mathbb{R}^3$ are only planes, round spheres or right circular cylinders.

Galina Filipuk (University of Warsaw)

On the Painlevé equations and discrete Painlevé equations

In this talk I shall present a few recent results on the solutions of the Painlevé equations and their discrete analogues.

Tatiana Firsova (Stony Brook University)

Pseudoconvexity and the Lambda Lemma

(joint work with Eric Bedford)

We relate the technique of filling totally real manifolds with boundaries in pseudoconvex domains to the holomorphic motion. That replaces the major technical part in the proof of lambda lemma by a transparent geometric argument.

Matthew Fleeman (University of South Florida)

Extremal domains for self-commutators acting on the Bergman space

In recent work, Olsen and Reguera have shown that Putnam's inequality for the norm of self-commutators can be improved by a factor of $\frac{1}{2}$ for Toeplitz operators with analytic symbol φ acting on the Bergman space $A^2(\Omega)$. This improved upper bound is sharp when $\varphi(\Omega)$ is a disk. In this talk we show that disks are the only domains for which the upper bound is attained.

Paul Gauthier (Université de Montréal)

Universal functions

In 1929 Birkhoff showed the existence of an entire function which is universal in the sense that its translates approximate all entire functions. In 1975 Voronin proved a spectacular theorem stating that the Riemann zeta-function exhibits a strong form of universality. In this context, Bagchi in 1982 gave a statement equivalent to the Riemann Hypothesis. Some universality results have been shown also in several complex variables.

Walter Hayman (Imperial College London)

Functions of locally bounded characteristic

In this talk I hope to show that a fairly wide class of functions meromorphic in the unit disk Δ behaves like

$$\exp\left\{(\alpha(\theta)+o(1))\,\frac{e^{i\theta}+z}{e^{i\theta}-z}\right\}$$

as z approaches the point $e^{i\theta}$ nontangentially from inside Δ outside a small exceptional set.

Nickolas Hein (University of Nebraska at Kearney)

Lower bounds in real Schubert Calculus

Let $\mathrm{Gr}(m,p)$ denote the Grassmaniann of m-planes in $(m+p)$-space. Boris Shapiro and Michael Shapiro made the surprising conjecture in 1993 that problems in the Schubert calculus of $\mathrm{Gr}(m,p)$ have all solutions real when their flags osculate a rational normal curve at real points. Sottile generated computational data which strongly supported this Shapiro Conjecture, and Eremenko and Gabrielov proved the conjecture in the special cases $m=2$ and $p=2$. The conjecture was eventually settled in the affirmative by Mukhin–Tarasov–Varchenko.

Eremenko and Gabrielov also studied a family of problems related to the Shapiro Conjecture by relaxing the reality condition on osculation points and restricting to Schubert problems given by an intersection of hypersurface Schubert varieties and perhaps a single Schubert variety of higher codimension. Their results were extended by Sottile and Soprunova to include Schubert problems involving two varieties of codimension greater than one. We discuss the generalization of this problem to general osculating Schubert problems whose solution sets are real varieties, and we discuss remarkable congruences and gaps in the possible numbers of real solutions to such problem. This is related to recent work of Mukhin and Tarasov.

V. V. Hemasundar Gollakota (South Indians' Welfare Society College)

Families of compact Riemann surfaces

The aim of the talk is to indicate how Koebe's General Uniformisation Theorem for planar Riemann surfaces may be used to construct families of compact Riemann surfaces of every genus in a very concrete way. It is also shown explicitly that in case of $g=1$ every compact Riemann surface of genus 1 occur in this family.

[1] L. Bieberbach, *Conformal Mapping.* Chelesa Publishing Company, New York, 1964.

[2] Gollakota V. V. Hemasundar, *Koebe's General Uniformisation Theorem for Planar Riemann surfaces*, Ann. Polon. Math. 100 (2011), 77-85.

[3] Kunihiko Kodaira, *Complex Analysis.* Cambridge University Press, New York, 2007.

[4] George Springer, *Introduction to Riemann surfaces*, Addison-Wesley Publishing Company, Inc., 1957.

Xavier Jarque (Universitat de Barcelona)

Wandering domains in Eremenko–Lyubich's class and Bishop's example

We briefly summarize some considerations on the existence and non-existence of wandering domains for entire maps, introduce Bishop's example of an oscillating wandering domain in Eremenko–Lyubich's class and give the main ideas to prove that there are no non-expected wandering domains in his example.

Wolf Jung (Gesamtschule Aachen-Brand)

Self-similarity of the Mandelbrot set

1) An overview is given of classic and recent self-similarity phenomena of the Mandelbrot set M: examples of homeomorphisms between subsets of M are shown, which are constructed by quasi-conformal surgery. And examples of asymptotic self-similarity are given, which shall mean that a sequence of rescaled subsets is converging to a model set.

2) A Misiurewicz point is a parameter in M, such that the corresponding quadratic polynomial is critically preperiodic. Fundamental domains in the parameter plane are constructed combinatorially; they are mutually homeomorphic and asymptotically similar as well.

3) The Misiurewicz point is approached by a sequence of small Mandelbrot sets with geometric scaling properties. The decorations of each copy are approximately similar to decorations of the corresponding small Julia sets. And the sequence of decorations has asymptotic model sets on multiple scales.

Agnieszka Kałamajska (University of Warsaw)

On one variant of interpolation inequality and its applications to the nonlinear eigenvalue problems

(joint work with Tomasz Choczewski and Jan Peszek)

It is well known [2] that the following inequalities hold for every $u \in C_0^\infty(\mathbb{R})$:

$$\int_{\mathbb{R}} |f'(x)|^p h(f(x))dx \le \left(\sqrt{p-1}\right)^p \int_{\mathbb{R}} \left(\sqrt{|f''(x)\mathcal{T}_h(f(x))|}\right)^p h(f(x))dx,$$

where $\mathcal{T}_h(\cdot)$ is certain transform of h. We are interested in the validity of their multidimensional variants

$$\int_{\Omega} |\nabla f(x)|^p h(f(x))dx \le C \int_{\Omega} \left(\sqrt{|\Delta f(x)\mathcal{T}_h(f(x))|}\right)^p h(f(x))dx,$$

where $u \in C_0^\infty(\Omega)$, Δ is the Laplace operator. The problem in n dimensions where $n > 1$ becomes much more complicated. We construct such inequalities, applying Hardy inequalities with best constants, as well as knowledge about constants for the vectorial Riesz transform. Applications to PDEs will also be discussed.

[1] A. Kałamajska, T. Choczewski, *On one variant of strongly nonlinear interpolation inequality with applications to nonlinear eigenvalue problems of elliptic type*, in preparation.

[2] A. Kałamajska, J. Peszek, *On some nonlinear extensions of the Gagliardo–Nirenberg inequality with applications to nonlinear eigenvalue problems*, Asymptotic Analysis 77 (2012), No. 3-4, 169–196.

Stanisława Kanas (University of Rzeszów)

Orthogonal polynomials and typically real functions related to generalized Koebe function

(joint work with A. Tatarczak)

The typically real functions $\mathcal{T}_{\mathbb{R}}$ [2, 3] consists of all analytic function satisfying a condition $\operatorname{Im} f(z) \operatorname{Im} z \geq 0$ in the unit disk. We will discuss geometric and analytic properties of the generalized typically-real functions defined via the generating function of Gasper type [1] related to the generalized Chebyshev polynomials of the second kind.

[1] G. Gasper, M. Rahman, *Basic hypergeometric series*, Cambridge University Press, 1997.

[2] M. S. Robertson, *On the coefficients of typically real functions*, Bull. Amer. Math. Soc. 41 (1935), 565–572.

[3] W. W. Rogosinski, *Über positive harmonische Entwicklungen und typische-reelle Potenzreihen*, Math. Z. 35 (1932), 93–121.

Thomas Kecker (University College London)

Local and global finite branching of solutions of ODEs in the complex plane

One aspect of the work presented is to show, for certain classes of ordinary differential equations, that all movable singularities of all solutions in the complex plane are either poles or algebraic branch points, i.e. the about any movable singularity z_0 a solution $y(z)$ can be represented by a Laurent series with finite principle part in a power or fractional power of $z - z_0$. The solutions of these equations extend, although they are locally finite-branched, in general over a Riemann surface with an infinite number of sheets. Another aspect of the work is to give conditions under which the equations admit solutions which are also globally finite-branched. For first-order equations this is covered by Malmquist's theorem for the case of algebroid solutions. We also give some examples of certain second-order equations admitting algebroid solutions.

Dmitry Khavinson (University of South Florida)

Boundary behaviour of universal power series

Consider a power series that converges on the open unit disk. It is called universal if its partial sums approximate arbitrary polynomials on all Mergelyan compact sets outside the open disk. A mergelyan compact set is a compact set with a connected complement. In this talk we will show that such series grow quite strongly and possess a Picard-type property near each boundary point of the unit disk.

Julia Koch (Julius-Maximilians-Universität Würzburg)

Dieudonné's Lemma for schlicht functions

Sharpening the Schwarz Lemma, Rogosinski's Lemma (1934) explicitly describes the value set $\{f(\zeta) : f \in B\}$ for the class $B := \{f : \mathbb{D} \to \mathbb{D} : f(0), f(0) \geq 0\}$. A few months ago, Roth and Schleißinger (2013) proved a schlicht version of Rogosinski's Lemma which showed that in this case, the value set has a beautiful geometry.

Dieudonné's Lemma (1931) states that, given any $0 \neq \zeta$, $\omega \in \mathbb{D}$, for a function from the class $B(\zeta, \omega) := \{f : \mathbb{D} \to \mathbb{D} : f(0) = 0, f(\zeta) = \omega\}$, the value set for the derivative at $z = \zeta$, $\{f(\zeta) : f \in B(\zeta, \omega)\}$ equals the closed disc of center $\frac{\omega}{\zeta}$ and radius $\frac{|\zeta|^2 - |\omega|^2}{|\zeta|(1-|\zeta|^2)}$. We try to follow the example stated above and describe the value set taken over the schlicht functions from the class $B(\zeta, \omega)$.

Aleksis Koski (University of Helsinki)

Subharmonicity results for energy-minimal mappings

Recently, Iwaniec and Onninen gave a new proof of the classical Radó–Kneser–Choquet theorem in the plane. The proof was based on the fact that the logarithm of the Jacobian determinant of a harmonic function is superharmonic, assuming that the Jacobian is positive. New computations have shown that even for solutions of more general Euler–Lagrange equations, there exist nonlinear differential expressions which are subharmonic (or superharmonic). This has yielded new knowledge of the solutions, for example, a generalization of the Radó–Kneser–Choquet theorem for p-harmonic mappings in the plane. We aim to give exposition to these results and classify the energy functionals which give rise to such differential expressions.

Janina Kotus (Warsaw University of Technology)

Metric entropy and stochastic laws of invariant measures for elliptic functions

We consider a class of *critically tame* elliptic function $f : \mathbb{C} \to \overline{\mathbb{C}}$. We give a construction of finite invariant measure absolutely continuous with respect to h-conformal measure for these maps, where where h is the Hausdorff dimension of the Julia set f. We establish the exponential decay of correlations, the Central Limit Theorem, and the Law of Iterated Logarithm with respect to the measure μ_h. We also prove that $h_\mu(f) < \infty$.

Jean-Jacques Loeb (Université d'Angers)

Smooth critical points of planar harmonic mappings

In this talk, inspired by a work of Lyzzaik, we present results on the local behaviour of germs of planar harmonic mapping. We introduce invariants associated to the complexified holomorphic map and some relations are given between them.

Alexander Logunov (St. Petersburg State University)

Several questions on harmonic functions in higher dimensions

We will discuss the higher dimensional analog of the Levinson "log log" theorem, the normal family conjecture for harmonic functions sharing the same zero set, and the gradient estimate of the ratios of harmonic functions.

http://arxiv.org/abs/1402.2888

Mikhail Lyubich (Stony Brook University)

Dynamics of dissipative polynomial automorphisms of $\mathbb{C}^2$

(joint work with Han Peters and Romain Dujardin)

Two-dimensional complex dynamics is a recent area of research that has some similarities with the classical one-dimensional theory but also exhibits essentially new phenomena that require new analytic and dynamical tools. We will discuss recent advances in this field for dissipative polynomial automorphisms of $\mathbb{C}^2$. They include, in particular, a nearly complete description of the dynamics on periodic Fatou components and exploration of the bifurcation locus in holomorphic families of maps in question.

Svitlana Mayboroda (University of Minnesota)

Localization of eigenfunctions and associated free boundary problems

The phenomenon of wave localization permeates acoustics, quantum physics, elasticity, energy engineering. It was used in construction of the noise abatement walls, LEDs, optical devices. Anderson localization of quantum states of electrons has become one of the prominent subjects in quantum physics, as well as harmonic analysis and probability. Yet, no methods predict specific spatial location of the localized waves.

In this talk I will present recent results revealing a universal mechanism of spatial localization of the eigenfunctions of an elliptic operator and emerging operator theory/analysis/geometric measure theory approaches and techniques. We prove that for any operator on any bounded domain there exists a "landscape" which splits the domain into disjoint subregions and indicates location, shapes, and frequencies of the localized eigenmodes. In particular, the landscape connects localization to a certain multi-phase free boundary problem, regularity of minimizers, and geometry of free boundaries.

Sabyasachi Mukherjee (Jacobs University Bremen)

On the topological differences between the Mandelbrot set and the tricorn

We study the iteration of quadratic anti-polynomials $\bar{z}^2 + c$ and their connectedness locus, known as the tricorn. The topology of the tricorn dif- fers vastly from that of the Mandelbrot set (the connectedness locus of quadratic polynomials); for example, the tricorn contains parabolic arcs, which are real- analytic arcs consisting of quasi-conformally equivalent parabolic parameters and there are bifurcations between hyperbolic components along parts of such arcs. Quite recently, Hubbard and Schleicher proved that the tricorn is not path-connected, confirming a conjecture of Milnor.

The goal of this talk is to prove the following results, which further elucidate the topological differences between these two parameter spaces.

1) Rational parameter rays at odd-periodic angles of the tricorn do not land at a single parameter, but accumulate on an arc of positive length in the parameter space. (This is a joint work with Hiroyuki Inou).

2) Centers of hyperbolic components are *not* dense on the boundary of the tricorn.

Shahar Nevo (Bar-Ilan University)

Differential polynomials and shared values

(joint work with Jürgen Grahl)

Let f and g be non-constant meromorphic functions in $\mathbb{C}$, a and b non-zero complex numbers and let n and k be natural numbers satisfying $n \geq 5k + 17$. Using Nevanlinna Theory, we show that if the differential polynomials $f^n + af^{(k)}$ and $g^n + ag^{(k)}$ share the value b counting multiplicities,then f and g are either equal or at least closely related.

Dan Nicks (University of Nottingham)

Superattracting fixed points of quasiregular mappings

(joint work with Alastair Fletcher)

This talk will consider iterative behaviour near fixed points of quasiregular mappings $f : \mathbb{R}^n \to \mathbb{R}^n$. Unlike the holomorphic case, non-injectivity of f near a fixed point x_0 is not enough to imply that x_0 is attracting. However, the iterates f^k will converge uniformly to x_0 on some neighbourhood if f is d-to-1 near x_0 and d exceeds the dilatation $K(f)$. One can then compare the rate of convergence for different points in the basin of attraction. A corollary is that, for a polynomial type quasiregular map with degree $> K(f)$, all points that escape to infinity (i.e. $f^k(x) \to \infty$ as $k \to \infty$) do so 'as fast as possible'.

Dmitry Novikov (The Weizmann Institute)

Multiplicity operators and non-isolated Noetherian intersections

Jani Onninen (Syracuse University/University of Jyvaskyla)

Existence of 2D traction free minimal deformations

In GFT we seek, as a generalization of the Riemann Mapping Problem, homeomorphisms that minimize certain energy integrals. No boundary values of such homeomorphisms are prescribed. This is interpreted as saying that the deformations are allowed to slip along the boundary, known as traction free problems. This leads us to determine the infimum of a given energy functional among homeomorphisms from X onto Y. Even in the basic case of the Dirichlet energy, it is certainly unrealistic to require that the infimum energy be attained within the class of homeomorphisms. Of course, enlarging the set of the admissible mappings can change the nature of the energy-minimal solutions. To avoid the Lavrentiev phenomenon one need to study monotone Sobolev mappings which is the subject of my talk.

Fedor Pakovich (Ben Gurion University)

On semiconjugate rational functions

A classification of commuting rational functions, that is of rational solutions of the functional equation $A(X) = A)$, was obtained in the beginning of the past century by Fatou, Julia, and Ritt. In the talk we will present a solution of a more general problem of description of semiconjugate rational functions, that is of rational solutions of the functional equation $A(X) = B)$ in terms of groups acting properly discontinuously on the Riemann sphere or complex plane.

Christopher Penrose (University of London)

Quasi-elementary correspondences

Katarzyna Pietruska-Pałuba (University of Warsaw)

Poincaré inequality on nested fractals

We will describe the notion of gradient on nested fractals (due to S. Kusuoka), and then we will derive the local Poincaré inequality in this setting. This inequality will be then used towards the definition of Poincaré–Sobolev spaces on those sets. We will discuss the relations of those spaces to other possible definitions of Sobolev spaces on nested fractals.

[1] K. Pietruska-Paluba, A. Stós, *Poincaré inequality and Hajłasz-Sobolev spaces on nested fractals*, Studia Math. 218 (2013), 1–26.

Mary Rees (University of Liverpool)

Parapuzzles in the neighbourhood of a hyperbolic component

The fundamental principle in dynamics of obtaining information about a parameter space, at least locally, from the dynamical plane of a map in the parameter space, is particularly useful in both transcendental and rational dynamics, where a linkage has developed between what have become known as puzzles and parapuzzles. A puzzle is essentially a sequence of successively finer Markov partitions, consisting of backward iterates of a "level zero" partition. A parapuzzle is similarly a sequence of successively finer partitions, wwith the level n partition of the parapuzzle derived from the level n partition of the puzzle. In some instances, notably the Yoccoz puzzle for quadratic polynomials and the puzzle for the exponential family which probably has its origins in the work of Devaney and Krych, the puzzle is essentially global in nature. The instance I want to focus on is a very free construction which works locally, in the neighbourhood of hyperbolic components in certain one-dimensional slices of rational maps. I want to look, in particular, at the correspondence between periodic points near a Fatou component. and hyperbolic components near a fixed hyperbolic component.

Lasse Rempe-Gillen (University of Liverpool)

Arc-like continua, the Eremenko–Lyubich class, and Eremenko's conjecture

In an article from 1989, Alex Eremenko was the first to systematically study the *escaping set* $I(f)$ of a transcendental function f. (This set consists of all points that converge to infinity under iteration of f.) In particular, he asked the following question, which has become one of the guiding problems of entire transcendental dynamics, and is now referred to as *Eremenko's Conjecture*:

If f is a transcendental entire function, is every connected component of the *escaping set* $I(f)$ unbounded?

I will discuss our current state of knowledge concerning this conjecture, and in particular try to explain what makes it so difficult. I shall focus mainly on the *Eremenko–Lyubich class* $\mathcal{B}$, defined by Alex and Misha Lyubich in the 1980s, where there is strong expansion along the escaping set.

It turns out that questions such as Eremenko's conjecture can be closely linked to topological properties of Julia sets of certain hyperbolic entire functions, and I will discuss recent results that describe the possible behaviour in this case. In particular, we shall see that every *arc-like continuum* (a large class of topological objects from classical continuum theory) can arise in the Julia set of a hyperbolic transcendental entire function.

Phil Rippon (The Open University)

Escaping boundary points of Baker domains

(joint work with Gwyneth Stallard)

A 'Baker domain' of a transcendental entire function is a periodic component of the Fatou set within which all points are escaping under iteration of the function. It is an open question whether the boundary of a Baker domain must contain at least one escaping point. We prove that for any univalent Baker domain the set of escaping boundary points forms a set of full harmonic measure with respect to the domain. The proof uses results about the boundary behaviour of conformal mappings and logarithmic capacity. We deduce a new sufficient condition for the escaping set to be connected.

Min Ru (University of Houston)

Quantitative geometric and arithmetic results for complement of divisors

In this talk, we introduce, for an effective divisor D on a smooth projective variety X, the notion of *Nevanlinna constant* of D, denoted by $\mathrm{Neva}(D)$. We then prove *the defect relation* $\delta_f(D) \leq \mathrm{Neva}(D)$ *for any Zariski-dense holomorphic mapping* $f : \mathbb{C} \to X$. Its arithmetic counterpart in Diophantine approximation is also obtained. The notion $\mathrm{Neva}(D)$, together with the result obtained, gives a unified description of the quantitative geometric and arithmetic properties of (X, D). It also recovers all known results along this direction simply by computing $\mathrm{Neva}(D)$ in various cases.

Sebastian Schleißinger (Julius-Maximilians-Universität Würzburg)

The chordal Loewner equation for slits

Recent progress in the theory of Loewner equations suggests that one of the most useful descriptions of a simple plane curve is by encoding it into a growth process modeled by the Loewner equation. We prove that any disjoint union of finitely many simple curves in the upper half-plane can be generated by the chordal multiple-slit Loewner equation with constant weights.

Nikita Selinger (Stony Brook University)

Classification of Thurston maps with parabolic orbifolds

In a joint work with M. Yampolsky, we give a classification of Thurston maps with parabolic orbifolds based on our previous results on characterization of canonical Thurston obstructions. The obtained results yield a partial solution to the problem of algorithmically checking combinatorial equivalence of two Thurston maps.

Mikhail Sodin (Tel Aviv University)

Zeroes of random and pseudo-random Taylor series

We describe a recent progress in understanding the distribution of zeroes of various classes of random and pseudo-random Taylor series. These results answer several questions left open by Littlewood and Offord, and by Kahane.

The talk is based on joint works with Alexander Borichev, Fedor Nazarov, and Alon Nishry, and also on a recent work by Ken Söze.

Frank Sottile (Texas A&M University)

Lower bounds for real solutions to systems of polynomials

Around 2000, as part of their work on the Shapiro conjecture, Eremenko and Gabrielov computed the topological degree of the real Wronski map. This was often positive, which established a lower bound for the number of real solutions to certain problems in the Schubert calculus. Since then, there have been many results establishing structure in the numbers of real solutions, particularly lower bounds, for polynomial systems with structure.

Gwyneth Stallard (The Open University)

Eremenko's conjecture on the components of the escaping set

(joint work with Phil Rippon)

Much of the recent work in transcendental dynamics has been motivated by Eremenko's conjecture that, for any transcendental entire function, there are no bounded components of the escaping set (the set of points that escape to infinity under iteration). By studying the components of the "fast escaping set" we are able to show that the escaping set is either connected or has uncountably many unbounded components. The proof uses properties of "Eremenko points" (constructed using Wiman–Valiron theory) together with a new covering lemma.

Norbert Steinmetz (TU Dortmund)

Sub-normal Solutions to Painlevé IV

(joint work with Christopher Classen)

The non-rational solutions to Painlevé's fourth equation

$$\text{(IV)} \qquad 2ww'' = w'^2 + 3w^4 + 8zw^3 + 4(z^2 - \alpha)w^2 + 2\beta$$

are meromorphic in the plane and satisfy

$$C_1 r^2 \leq T(r, w) \leq C_2 r^4,$$

where T denotes the Nevanlinna characteristic.

Solutions satisfying $T(r,w) = O(r^2)$, and hence

$$T(r,w) \asymp r^2,$$

are called *sub-normal.* For example, the solutions to any of the Weber–Hermite equations

$$\text{(WH)} \qquad w' = -2(1+\alpha) + 2zw + w^2 \quad \text{and} \quad w' = -2(1-\alpha) - 2zw - w^2$$

are sub-normal solutions to equation (IV) with $\beta = -2(1 \pm \alpha)^2$.

Repeated application of the Bäcklund transformations

$$\text{(BT)} \qquad w \mapsto \frac{w' - \sqrt{-2\beta} - 2zw - w^2}{2w} \quad \text{and} \quad w \mapsto -\frac{w' + \sqrt{-2\beta} + 2zw + w^2}{2w}$$

to any solution to (WH) leads to a solution to (IV), of course with new parameters. These solutions, named after *Weber–Hermite*, are also sub-normal. It will be proved that the converse also holds:

Theorem. *Any sub-normal solution to* (IV) *is also a Weber–Hermite solution.*

Vitaly Tarasov (Indiana University–Purdue University Indianapolis)

Lower bounds for numbers of real solutions in problems of Schubert calculus

I will discuss lower bounds for the numbers of real solutions of real Schubert calculus problems in Grassmannians related to osculating flags. Equivalently, those numbers are the numbers of real monic Fuchsian differential equations that have the trivial monodromy with the prescribed data for the equations being singular points and the exponents at singular points (the indicial equations). The strong form of the remarkable Shapiro–Shapiro conjecture, proved in 2007 by Mukhin, Varchenko and VT, states that a monodromy-free monic Fuchsian differential equation with real singular point is real itself, and each such an equation occurs with multiplicity one. The proof was done by interpreting the Schubert calculus problem as the eigenvalue problem for a family of commuting operators, called Gaudin Hamiltonians. In the case of real singular points those operators are given by real symmetric matrices and thus have real eigenvalues. By a minor modification this method produces also lower bounds for the number of monodromy-free monic Fuchsian differential equations such that the set of singular points together with the exponents is invariant under the complex conjugation. The bounds are expressed via coefficients of products of Schur polynomials. The bounds are tight in surprisingly many cases, although there are some cases when the bounds are known not being tight.

Alexander Volberg (Michigan State University)

Removable sets of Lipschitz solutions of (fractional) Laplace equation $(-\Delta)^a u = 0$

(joint work with V. Eiderman and F. Nazarov)

We consider Lipschitz in $\mathbb{R}^n$ functions solving the above mentioned equation with $1/2 < a \leq 1$ or $a = n/2$ in $\mathbb{R}^n \setminus E$. We consider only Lipschitz in $\mathbb{R}^n$ solutions. We show that the singular set E must have special geometry.

Katsutoshi Yamanoi (Tokyo Institute of Technology)

Nevanlinna theory for holomorphic curves into algebraic varieties of maximal albanese dimension

A projective variety X is called of maximal albanese dimension if the dimension of X is equal to the dimension of the image $a_X(X)$ of the albanese morphism $a_X : X \to \mathrm{Alb}(X)$. These varieties consist of (1) abelian varieties, (2) sub-varieties of abelian varieties, and (3) generically finite coverings of varieties of (1) and (2). We shall discuss the second main theorem in Nevanlinna theory for holomorphic curves $f : Y \to X$ from finite ramified covering space $Y \to \mathbb{C}$ to varieties X of maximal albanese dimensions. As an application, when X is of maximal albanese dimension and of general type, we shall discuss pseudo-algebraic hyperbolicity of X.

Anna Zdunik (University of Warsaw)

Inducing schemes in holomorphic dynamics

I will outline an efficient inducing scheme, which allows to study a family of equilibrium measures for rational maps in the Riemann sphere. Several results (rigidity theorems, refined geometry of equilibrium measures, stochastic laws, no phase transitions) will be discussed.

Posters

Vasiliki Evdoridou (The Open University)

Sufficient conditions for a point to be fast escaping

Let f be a transcendental entire function. The set of points that eventually escape to infinity faster than the iterates of the maximum modulus function is known as the fast escaping set and has several nice properties. Rippon and Stallard showed that, if the maximum modulus satisfies a certain regularity condition, then points which eventually escape faster than the iterates of some smaller function than the maximum modulus are actually fast escaping. We are interested in finding similar conditions. In particular, we prove that under a stronger regularity condition, points which eventually escape to infinity faster than the iterates of an even smaller function are actually fast escaping.

Khudoyor Mamayusupov (Jacobs University Bremen)

Parabolic Newton maps and surgery

If you start with an entire map $p(z)\exp(q(z))$ then its Newton's map $N_{p\exp(q)}(z) := z - p(z)/(p'(z)+p(z)q'(z))$ is a rational map, all roots of a polynomial $p(z)$ become (super)attracting fixed points of its Newton map and the Newton map has a repelling or a parabolic fixed point at infinity depending on polynomial $q(z)$. These are all possible entire functions whose Newton's method are rational maps. One can do a parabolic surgery by P. Haissinsky to change repelling fixed point at infinity (when $q(z) = \text{const}$) to make it parabolic and gets another rational map which turns out to be again a parabolic Newton map. This surgery defines a map from a set of hyperbolic postcritically finite Newton maps to a set of stable postcritically minimal parabolic Newton maps. This correspondence is nice in a sense that it is bijective and preserves dynamics on respective Julia sets.

David Martí Pete (The Open University)

Annular itineraries for $\mathbb{C}^*$

We are interested in studying the different rates of escape of points under iteration by transcendental holomorphic self-maps of $\mathbb{C}^*$. We do so by comparing them with the iterated maximum and minimum modulus functions. Using an annular covering lemma we are able to construct different types of orbits, giving fast escaping and arbitrarily slowly escaping points to either 0, infinity or both of them, and points with periodic itineraries as well. These results are analogous to the ones that Phil Rippon and Gwyneth Stallard recently proved for entire functions. For any kind of escaping itinerary, the corresponding set of fast escaping points is always non-empty, its boundary equals the Julia set and all its connected components are unbounded.

John Osborne (The Open University)

Connectedness properties of the set where the iterates of an entire function are unbounded

(joint work with Phil Rippon and Gwyneth Stallard)

We investigate some connectedness properties of the set of points $K(f)^c$ where the iterates of an entire function f are unbounded. In particular, we show that $K(f)^c$ is connected if iterates of the minimum modulus of f tend to ∞. For a general transcendental entire function, we show that $K(f)^c \cup \{\infty\}$ is always connected, and that $K(f)^c$ is either connected or has uncountably many components. We also give a number of new results on iterating the minimum modulus of an entire function which are of independent interest.