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Institute of Mathematics, Polish Academy of Sciences
Warsaw Center of Mathematics and Computer Science

School and Conference

ANALYTIC, ALGEBRAIC AND GEOMETRIC ASPECTS OF DIFFERENTIAL EQUATIONS

September 6 – 19, 2015

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The Banach Center Conference

Analytic, Algebraic and Geometric Aspects of Differential Equations

September 14 – 19, 2015

Abstracts of lectures and talks

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Takashi Aoki (a collaboration with Mika Tanda and T.Takahashi)
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The hypergeometric function and WKB solutions

The hypergeometric function is a well-known classical object which plays a role in the study of various fields not only of mathematics but of physical sciences. We introduce a large parameter in the Gauss hypergeometric differential equation and consider formal solutions which are called WKB solutions of it. They can be constructed easily and it is known by Koike and Schäfke that they are Borel summable in suitable domains. We investigate the relation between the hypergeometric function and the Borel resummed WKB solutions. As an application, the asymptotic behavior of the hypergeometric function with respect to the parameter are obtained.

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Dennis Bacani (joint work with H. Tahara and M. A. C. Tolentino)
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Lifespan of solutions to nonlinear Cauchy problems with small analytic data

We consider the lifespan of solutions to Cauchy problems for nonlinear analytic partial differential equations with small analytic data. We show that the lifespan of the solution becomes longer as the initial data become smaller and its dependence on the smallness of the data can be sharply described by the property of the equation.

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Yulia Bibilo (joint work with G. Filipuk)
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Inverse monodromy problems and middle convolution

We apply additive and multiplicative middle convolution to the Riemann–Hilbert problem and to isomonodromic deformations. We present the scheme generating constructive solutions to the Riemann–Hilbert problem [1] and show corresponding examples. Also it is known that middle convolution operation preserves Schlesinger's deformation equations for non-resonant Fuchsian systems. We give examples when middle convolution preserves and also does not preserve Bolibruch's non-Schlesinger deformations of resonant Fuchsian systems [2], [3].

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Value distribution and growth of solutions of the second order ODE's

Complex differential equations have been analysed from Nevanlinna theory point of view since the beginning of the theory itself in the 1920's. As early as 1933 K. Yosida proved the theorem of J. Malmquist applying methods of Nevanlinna theory. The first systematic application of the theory to solutions of differential equations was conducted in 1940's by H. Wittich and this global approach gained popularity in the 1970's. Major contributions here belong, among others, to: S. Bank, J. Clunie, G. Gundersen, A. Hinkkanen, G. Frank, I. Laine, J. Langley, A.Z. Mohon'ko and V.D Mohon'ko, J. Rossi and N. Steinmetz.

Value distribution and growth of meromorphic solutions of second order differential equations in general, and of equations with the Painlevé property in particular, have also been a subject of thorough study. Thus it has been possible to estimate growth order, distribution of zeros, the amount of exceptional values, Nevanlinna defects, ramification indices and other parameters of transcendental meromorphic solutions. There are still some open problems, however, for instance improving deficiency and ramification results both for constants and for small target functions, or establishing estimates of deviations and the structure of the set of exceptional values in the sense of Petrenko.

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Semi-classical orthogonal polynomials and the Painlevé equations

In this talk I shall discuss semi-classical orthogonal polynomials arising from perturbations of classical weights. It is shown that the coefficients of the three-term recurrence relation satisfied by the polynomials can be expressed in terms of Wronskians which involve special functions. These Wronskians are related to special function solutions of the Painlevé equations. Using this relationship recurrence relation coefficients can be explicitly written in terms of exact solutions of Painlevé equations.

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Ellen Dowie (joint work with P. A. Clarkson)
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Rational solutions of the Boussinesq equation and the non-linear Schrödinger equation

Rational solutions of the Boussinesq equation give rise to water wave solitons, by examining the form of these solutions and considering the behaviour of the roots, the aim is to establish the behaviour of this family of solutions. In particular, the solutions considered are the second logarithmic derivative of polynomial functions in x and t of symmetric degree $n(n+1)$. Solutions have been found up to $n=5$.

Investigation into both the Boussinesq and the non-linear Schrödinger equation will be discussed along with complex root and solution plots.

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Summability of formal solutions of q-difference equations and confluence

When we consider linear differential systems of the complex variable, formal power series appear as solutions. The series diverge, but we have solutions, which are analytic in some sector and Gevrey asymptotic to the formal series. The fact that various asymptotic solutions do not glue to a single solution on the Riemann surface of the logarithm is called the Stokes phenomenon. In this talk, we will explain how we may approach the asymptotic solution using q-deformations. We will replace the differential system $Y'(z) = A(z)Y(z)$ by the q-difference system $Y(qz) = Y(z) + (q-1)A(z)Y(z)$, where $q > 1$ is a real number, and see what happen when q goes to 1.

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Special values of hypergeometric series

In this talk, we present a new method for finding identities for hypergeoemtric series, such as the (Gauss) hypergeometric series, the generalized hypergeometric series and the Appell-Lauricella hypergeometric series. Furthermore, we demonstrate this method; we will see some identities for hypergeometric series. For example, we can get the following identity for Appell hypergeometric series:

$$F_1\left(\begin{matrix}3\alpha, 2\beta-1, 2\alpha-2\beta+1\\ 2\alpha+\beta\end{matrix}; \frac{3}{4}, \frac{1}{4}\right) = \frac{2^{4\alpha+1}\pi\Gamma(2\alpha+\beta)}{3^{3\alpha+1}\Gamma(\alpha+1/3)\Gamma(\alpha+2/3))\Gamma(\beta)}.$$

The above says that Appell hypergeometric series under appropriate conditions can be expressed in terms of Gamma functions. However, such identity for Appell series, that is, hypergeometric series with several variables, have never been obtained. Therefore, our method will become a powerful tool for investigating hypergeometric identities for many kinds of hypergeometric series, especially for Appell-Lauricella hypergeometric series.

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On $(q;h)$-Weyl algebras

We introduce $(q;h)$-deformation of the Weyl algebra and study the ladders in this algebra, which give the factorization of certain q- and h-difference operators of second order. We also show that the q-deformed universal enveloping algebra $U_q(sl(2,\mathbb{C}))$ is embedded into the tensor product of two $(q;h)$-Weyl algebras. The results are presented in [1] and [2].

References

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A Maillet type theorem for generalized power series

We consider an ordinary differential equation

$$F(z,u,\delta u,...,\delta^m u)=0$$

of order m, where $F(z,u_0,u_1,...,u_m)\neq 0$ is a polynomial of $m+2$ variables and $\delta = z\frac{d}{dz}$. According to Maillet's theorem, if a formal power series $\varphi=\sum_{n=0}^{\infty} c_n z^n \in \mathbb{C}[[z]]$ satisfies such an equation, then there is $k>0$ such that the power series $\varphi=\sum_{n=0}^{\infty}\frac{c_n}{n!^{1/k}}z_n$ converges in some neighborhood of zero.
In the talk we propose an analogue of this theorem for generalized power series of the form

$$\varphi=\sum_{n=0}^{\infty} c_n z^{s_n}$$

whose power exponents s_n satisfy $\lim_{n\to\infty}\operatorname{Re} s_n=+\infty$.

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Connection problem for regular holonomic systems

In this talk, we shall propose a formulation of a connection problem for regular holonomic systems. In several works, connection problems for some particular regular holonomic systems are solved. For example, J. Sekiguchi (RIMS Kokyuroku, 773 (1991), 66-77) and M. Kato (Kyushu J. Math., 66 (2012), 325-363) studied connection problems for Appell's hypergeometric function F_2. They formulated the connection problem as a connection problem between two sets of solutions each of which is defined at an intersection point of irreducible divisors of the singular locus. We formulate the connection problem in another way.

We consider that the connection problem is based on the action of the local monodromies. For regular holonomic systems, the local monodromy is defined for each irreducible divisors of the singular locus. Then, if we want to go in an intrinsic way, we should formulate the connection problem as a connection problem between irreducible divisors. We also take into account of the results of R. Gérard (J. Math. Pures et Appl., 47 (1968), 321-404) and of M. Yoshida and K. Takano (Funkcial. Ekvac., 19 (1976), 175-189), both of which give canonical local solutions at a normally crossing point of irreducible divisors.

Under our formulation, we can solve connection problems for various regular holonomic systems such as Appell's F_1 and F_2, the hypergeometric function on the Grassmannian manifold $G_{3,6}$, and so on.

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$(q;h)$-deformation of $U(sl(2))$

We introduce the so called $(q;h)$-deformed Weyl algebra and study various algebraic features appearing in this algebra. Next we shall show that the q-deformed universal enveloping algebra (quantum group) $U_q = U_q(sl(2))$ is embedded into the tensor product of two $(q;h)$-Weyl algebras. Then we will analyze the so called structure ladder of U_q by means of ladder theory. It will turn out that many important properties of this algebra are preserved under the $(q;h)$-deformation. We will point out the relation to the angular momentum part of the Hydrogen atom.

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On the relationship between the BNR equation and the Pearcey system

The BNR equation

$$\left(\eta^{-3}\frac{d^3}{dx_1^3}+\frac{1}{2}c\eta^{-1}\frac{d}{dx_1}+\frac{1}{4}x_1\right)\psi=0$$

is an important example in the exact WKB analysis for higher-order ordinary differential equation and is obtained by restricting the Pearcey system

$$\begin{cases}\left(\eta^{-3}\dfrac{\partial^3}{\partial x_1^3}+\dfrac{1}{2}x_2\eta^{-1}\dfrac{\partial}{\partial x_1}+\dfrac{1}{4}x_1\right)\psi=0,\\ \left(\eta^{-1}\dfrac{\partial}{\partial x_2}-\eta^{-2}\dfrac{\partial^2}{\partial x_1^2}\right)\psi=0\end{cases}$$

to $x_2=c$.

In this talk, we will discuss the relationship between the BNR equation and the Pearcey system in the exact WKB analysis, and explain that this relationship can be understood as a quantization of a versal unfolding.

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k-summability of formal solutions for certain partial differential equations with time dependent polynomial coefficients

We study the k-summability of divergent formal solutions for the Cauchy problem of linear partial differential equations with time dependent polynomial coefficients. Under some assumptions for the operator, we will give a result of the k -summability of formal solutions in terms of the global analyticity and the exponential growth estimate of the Cauchy data.

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Strongly regular sequences, proximate orders and summability

Flat functions in Carleman ultraholomorphic classes, defined in terms of a strongly regular sequence $M = (M_p)_{p\in N_0}$, have been constructed in optimal sectors (see [2]). This fact is the key for obtaining kernels of summability (in the sense of W. Balser) in this general situation, see [1]. However, these achievements are available only if the sequence induces a Lindelöf proximate order. The purpose of this talk is to show that this is always the case. As a consequence, and by means of the characterization of strongly regular sequences in terms of regular variation, we prove that the growth index $\gamma(M)$ defined by V.Thilliez [3] and the order of quasianalyticity $\omega(M)$ introduced by J. Sanz [2] are indeed equal.

References

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Convolutions and analytical continuability of holomorphic functions

In the resurgent analysis, the following subspace $\mathscr{R}$ of 1-Gevrey formal series plays an essential role: $\widehat{\varphi} = \sum_{j=0}^{\infty} \varphi_j z^j \in \mathscr{R}$ if its formal Borel transform

$$B(\widehat{\varphi})(\zeta) = \varphi_0\delta + \varphi_B(\zeta),$$
$$\varphi_B(\zeta) = \sum_{j=1}^{\infty} \varphi_j \frac{\zeta^{j-1}}{(j-1)!} \in \mathbb{C}(z)$$

is endlessly continuable, i.e., for all $L > 0$, there is a finite subset $\Omega_L \subset \mathbb{C}$ such that $\varphi_B(\zeta)$ can be analytically continued along every path of length less than L avoiding Ω_L. We call such formal series resurgent formal series.
Via the Borel transformation, the ring structure of $\mathbb{C}[[z]]$ induces the following product structure on $\mathbb{C}\{\zeta\}$, For $f(\zeta), g(\zeta) \in \mathbb{C}\{\zeta\}$, the convolution product $f * g(\zeta)$ of

them is given by

$$f * g(\zeta) = \int_0^\zeta f(\zeta')g(\zeta')d\zeta'.$$

However, in general, describing the singularity structure of $f * g(\zeta)$ is not so easy. Recently, D. Sauzin gave precise estimates for $f_1 * \cdots * f_n$, where $f_1, \ldots ; f_n$ are holomorphic functions analytically continuable along any path avoiding discrete subset $\Omega \subset \mathbb{C}$ that is stable under addition. (See [3] and [4].) Further, in [1], they discussed singularity structure of convolution products of general endless continuable functions.

In this talk, we develop their theory and discuss the analytic properties of $f_1 * \cdots * f_n$ for general endless continuable functions $f_1, \ldots ; f_n$. As a consequence, we obtain the following

Theorem. *If* $\widehat{\varphi} \in \mathscr{R} \cap z\mathbb{C}[[z]]$ *and* $F(w) \in \mathbb{C}\{w\}$*, then* $F(\widehat{\varphi}) \in \mathscr{R}$*.*

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Ramified irregular singularities of meromorphic connections and plane curve singularities

In this talk I will propose similarity between ramified irregular singularities of meromorphic connections on formal disk and plane curve singularities. First I will relate Komatsu-Malgrange irregularities of meromorphic connections to intersection numbers and Milnor numbers of plane curve germs. Next we will see that local Fourier transforms of connections can be seen as blow up of plane curves. Moreover a necessary and sufficient condition for an irreducible connection to have a resolution of the ramified singularity is determined as an analogy of the resolution of plane curve singularities. Finally, for meromorphic connections, I will define an analogue of Puiseux characteristics which are topological invariants of plane curve singularities and show that it can be seen as an invariant of Stokes structures of meromorphic connections.

References

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Semi classical orthogonal polynomials and general Schlesinger system

In this talk we study the relation between semi-classical orthogonal polynomials and nonlinear differential equations coming from the isomonodromic deformation of linear system of differential equations on $\mathbb{P}^1$. There are many works establishing this kind of relations between semi-orthogonal polynomials with the weight functions taking from the integrands for hypergeometric, Kummer, Bessel, Hermite, Airy integrals and Painlevé equations. We discuss some extension of these results for the semi-classical orthogonal polynomials with the weight functions coming from the general hypergeometric integrals on the Grassmannian $G_{2,N}$. To establish the desired relations, we make use of the Atiyah-Ward Ansatz construction of particular solutions for the 2×2 Schlesinger system and its degenerated ones.

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On the center manifold of unfolded complex saddle-node singularities

It is well known that a system of analytic ODE's corresponding to a center manifold of a saddle-node singularity possesses a unique formal solution, which, while in general divergent, can be associated true analytic solutions on certain sectors of the complex plane by the Borel summation. In this talk, I will show how the formal and the sectoral solutions unfold under a small perturbation that separates a double (irregular) singularity into two simple (regular) ones, and explain the Stokes phenomenon via confluence.

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Bartosz Langowski
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On Sobolev and potential spaces related to Jacobi expansions

We define and study Sobolev spaces associated with Jacobi expansions. We prove that these Sobolev spaces are isomorphic, in the Banach space sense, with potential spaces (for the Jacobi 'Laplacian') of the same order. This is an essential generalization and strengthening of the recent results [1] concerning the special case of ultraspherical expansions, where in addition a restriction on the parameter of type was imposed. We also present some further results and applications, including a variant of Sobolev embedding theorem. Moreover, we give a characterization of the Jacobi potential spaces of arbitrary order in terms of suitable fractional square functions. As an auxiliary result of independent interest we prove L^p-boundedness of these fractional square functions.

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Multi-level Gevrey solutions of singularly perturbed linear partial differential equations

We study the asymptotic behavior of the solutions related to a family of singularly perturbed linear partial differential equations in the complex domain. The analytic solutions obtained by means of a Borel- Laplace summation procedure are represented by a formal power series in the perturbation parameter. Indeed, the geometry of the problem gives rise to a decomposition of the formal and analytic solutions so that a multi-level Gevrey order phenomenon appears. This result leans on a Malgrange-Sibuya theorem in several Gevrey levels.

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Gevrey solutions of a singular integro-differential equation

Let $(t,x) \in \mathbb{C}_t \times \mathbb{C}_x^n$ and fix positive integers m, q and d. We consider the equation

$$(t\partial_t)^m u = F(t, x, \{\partial_t^{-a}(t\partial_t)^b \partial_x^\alpha u\}_{(a,b,\alpha)\in\Lambda}),$$

where the index set Λ is given by $\{(a,b,\alpha) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N}^n : \ a \leq q;\ b + \lceil d|\alpha| \rceil \leq m + a \text{ and } b \leq m\}$. Without the operator ∂_t^{-1}, this is nothing but singular partial differential equation of Gérard and Tahara. This equation also includes, in essence, the system of equations considered by Bielawski in his investigations on Calabi metrics. Using the Fixed-point Theorem, we prove that if the coefficients of the partial Taylor expansion of F are holomorphic in t and of Gevrey order d in x, then this equation admits a unique solution $u(t,x)$ having the same properties.

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Mean values and real analytic functions

In the first part we introduce integral mean value functions which are averages of integral means over spheres/balls and over their images under the action of a discrete group of complex rotations. In the case of real analytic functions we derive higher order Pizzetti's formulas. As applications we obtain a maximum principle for polyharmonic functions and a characterization of convergent solutions to higher order heat equations $\partial_t u = \Delta^l u$.

In the second part we derive a characterization of real analytic functions in terms of integral means over balls. The characterization justifies introduction of a definition of analytic functions on metric measure spaces.

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Summability of divergent solutions of some linear partial differential equations with variable coefficients

In this talk I will consider the Cauchy problem for some linear partial differential equations of two complex variables $(t, z) \in \mathbb{C}^2$ with variable coefficients with respect to t. I will give sufficient conditions for the summability of formal power series solutions in terms of properties of the inhomogeneity and the initial data.

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Exponential growth order of solutions for Moser irreducible system and a counterexample for Barkatou's conjecture

We study the maximal exponential growth order of solutions of the following singular system of ordinary differential equations of apparent Poincaré rank $p \geq 1$ under the Moser irreducibility condition,

$$Ly = 0, \quad L = (p, A(z)) := z^{p+1}(d/dz)I_N - A(z), \tag{1}$$

where $A(z) \in M_N(\mathbb{C}\{z\})$ with $A(z) = \sum_{n=0}^{\infty} A_n z^{k+n}$, $k = O(A) \geq 0$ (order of zeros), $r = \operatorname{rank} A_0 \geq 1$.

In 1960, J. Moser defined the notion of irreducibility as follows. Let define two numbers $m(A)$ and $\mu(A)$ by

$$m(A) = p - k + r/N,$$
$$\mu(A) = \min_{P(z) \in GL_N(\mathbb{K}[z])} \left\{ m(A_P)\ ;\ A_P := P^{-1}AP - z^{p+1}P^{-1}P'(z) \in M_N(\mathbb{C}\{z\}) \right\}.$$

Then, in the case when $m(A) > 1$, he called the operator L reducible if $m(A) > \mu(A)$, and irreducible otherwise. He characterized the irreducibility condition in the following way; *The operator $L = (p, A(z))$ is irreducible if and only if*

$$\mathscr{P}_A(\lambda) := \left[z^r \times \left(\det(\lambda I_N - A(z)/z^{k+1} \right) \right]_{z=0} \not\equiv 0.$$

We denote by $\rho(L)$ the maximal exponential growth order of solutions of the equation (1). Then we can prove the following estimate for $\rho(L)$; *Let the operator $L = (p, A(z))$ be Moser irreducible with a nilpotent constant term $A(0) = A_0$. Then $\rho(L)$ is estimated by*

$$p - \frac{N - d - r}{N - d} \leq \rho(L) \leq p - \frac{1}{k_1}, \quad d = \deg_\lambda \mathscr{P}_A(\lambda),\ ()\mathrm{k}_1 = \min\{\mathrm{k}\ ;\ \mathrm{A}_0^{\mathrm{k}} = \mathrm{O}\}.$$

The best possibility of this estimate will be shown by an example. We remark that the first inequality is found in his lecture by M. Barkatou without proof [p. 31, see below].

Of course, it is desirable to specify $\rho(L)$ exactly, if possible. M. Barkatou stated a result on this problem in the lecture under some condition [p. 33], and gave a conjecture that; *For Moser irreducible operator $L = (p, A(z))$ it may hold that*

$$\rho(L) = p - s_0(A),$$

$$s_0(A) := \min_{1 \le j \le N} \frac{O(p_j)}{j}, \quad p_A(\lambda, z) = \det(\lambda I_N - A(z)) = \sum_{j=0}^{N} p_j(z) \lambda^{N-j},$$

in the tutorial lecture at the conference ISSAC'10 in Münich [p. 35]. We note that in the case when $s_0(A) = 0$ the assertion is trivial since $s_0(A) = 0$ if and only if the constant term A_0 is not nilpotent.

In the lecture, we give a counterexample to this conjecture, and present some properties of system transformation which makes change the Jordan canonical form of A_0 in Moser irreducible case.

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Difference reducibility of linear difference equations

Wallenberg [2] considered common solutions of both a linear differential equation of 2nd-order and an algebraic differential equation of 1st-order over the rational function field $\mathbb{C}(x)$. He showed that if the latter equation has at least two different homogeneous terms except for the constant term, then every common solution is Liouvillian over $\mathbb{C}(x)$. In this talk, we give a difference analogue of his result of higher-order.

A pair $\mathcal{K} = (K, \sigma)$ is called a difference field if K is a field and $\sigma : K \to K$ is an injective endomorphism. For difference fields $\mathcal{L} = (L, \sigma_L), \mathcal{K} = (K, \sigma_K)$, we call $\mathcal{L}/\mathcal{K}$ a difference extension if L/K is a field extension and $\sigma_L|_K = \sigma_K$. Let $\mathcal{L}/\mathcal{K}$ be a difference extension. We say a linear difference equation of nth-order over $\mathcal{K}$ is difference reducible in $\mathcal{L}$ if at least one of its solutions in $\mathcal{L}$ satisfies some algebraic difference equation of order less than n over $\mathcal{K}$. Difference reducibility is an analogue of differential reducibility [1]. We show that every linear difference equation over $\mathcal{K}$ being difference reducible in $\mathcal{L}$ has a solution satisfying an algebraic difference equation of special form. As applications of the result, we will show difference irreducibility of some linear q-difference equations.

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On coupling equations and their reversibility

The notion of coupling equation was introduced by Tahara (2007, Publ. RIMS, **43**), where he studied the transformations between solutions to two nonlinear partial differential equations in a complex domain. The explicit form of the coupling equation is

$$\frac{\partial \phi}{\partial t} + \sum_{m\geq 0} D^m[F](t, x, z_0, z_1, \ldots, z_{m+1}) \frac{\partial \phi}{\partial z_m} = G(t, x, \phi, D[\phi]).$$

where $D = \partial/\partial x + \sum_{i\geq 0} z_{i+1}\partial/\partial z_i$ denotes the formal vector field, and $\phi(t, x, z_0, z_1, \ldots)$ is the unknown function, which was actually treated as a formal power series of a special form in infinitely many variables $(t, z) = (t, z_0, z_1, \ldots)$.

It would be desirable, from several aspects, to give a framework based on a notion of holomorphic functions in (t, x, z), or that of functions continuous in t and holomorphic in (x, z).

In this talk, we report our recent study on coupling equations regarded as functional equations, especially on their solvability and their reversibility.

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Hypergeometric functions with several variables

Starting from the study of rigid local systems by N. Kats and Yokoyamas work on rigid Okubo systems, we now have a new understanding of Fuchsian linear ordinary differential equations as in [2]. The operations introduced by Katz are extended by Haraoka [1] to the case of certain holonomic systems. We have classified rigid ordinary differential equations by their spectral types. If the rigid equations have more than 3 singular points, we can intro- duce new variables as the coordinate of the moduli space of singular points and then we have infinitely many hypergeometric equations with several variables including Appells $F1 \sim F4$. We can apply the results in [2] to these general hypergeometric equations to the study of their rigidity, irreducibility, contiguous relations, integral representations, series expansions etc.

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The Stokes phenomenon in certain partial differential equations

We consider the well-known fact (called the Stokes phenomenon) that the formal solution of PDE can have different asymptotic expansions in different sectors of the complex plane. We focus our attention to derive the Stokes lines and anti-Stokes lines for the heat equation. Based on the obtained results we apply them for generalizations of the heat equation with meromorphic initial conditions. It is worth pointing out that the fundamental tool that we use to formulate the main theorems is the Borel k-summability.

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Multi-level asymptotics for singularly perturbed linear partial differential equations with ultraholomorphic coefficients

We study the asymptotic behavior of solutions to a family of singularly perturbed partial differential equations in the complex domain, with coefficients admitting a representation in terms of elements of general ultraholomorphic classes in a sector. These analytic solutions are asymptotically represented by a formal power series in the perturbation parameter. The geometry of the problem and the nature of the elements involved in it give rise to different asymptotic levels, related to the so-called strongly regular sequences defining the classes under consideration. The result, which generalizes a previous one by A. Lastra and S. Malek [1], rests on a novel version of a multi-level Ramis-Sibuya theorem and on the summability procedures introduced in [2].

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A generalisation of WDVV equation and flat structures

WDVV equations appeared originally in physical papers on two-dimensional field theory and play a central role in the construction of Frobenius-Saito structures (cf. [1], [5]). The purpose of this talk is to introduce the notion of a generalised WDVV equation, formulate one of generalisations of ordinary differential equations of Okubo type to several variables and explain applications of them to the construction of flat structures of discriminants of complex reflection groups (an analogue of K. Saito's result on real reflection groups [6], [7]) and flat structures associated to algebraic solutions to Painlevé VI (an analogue of the work of Dubrovin and Mazzocco [2]). This talk is based on the joint work with M. Kato and T. Mano (cf. [3], [4]).

We start with introducing weighted homogeneous polynomials $h_1(x), h_2(x), \dots, h_n(x)$ $(x = (x_1, x_2, \dots, x_n))$ such that $Eh_j = (w_j + w_n)h_j \quad (j = 1, 2, \dots, n)$ and that

$$h_j = \begin{cases} x_j x_n + h_j^{(0)}(x_1, \dots, x_{n-1}) & (j = 1, 2, \dots, n-1), \\ \dfrac{1}{2}x_n^2 + h_n^{(0)}(x_1, \dots, x_{n-1}) & (j = n) \end{cases}$$

with functions $h_j^{(0)}(x_1, \dots, x_{n-1})$ of $x' = (x_1, \dots, x_{n-1})$, where $E = \sum_{j=1}^n w_j x_j \partial_{x_j}$ is an Euler vector field with non-zero constants $w_1, \dots, w_n$. Using $h_j(x)$ $(j = 1, 2, \dots, n)$, we define an $n \times n$ matrix C such that $C_{ij} = \partial_{x_i} h_j (= \text{the}(i, j)\text{entry of} C)$. It is easy to see that $C_{nj} = x_j$ $(j = 1, 2, \dots, n)$. We define matrices $\tilde{B}^{(p)} = \partial_{x_p} C$ $(p = 1, 2, \dots, n)$ and $T = \sum_{j=1}^n w_j \tilde{B}^{(j)}$. Then the totality of relations of matrix entries of $\tilde{B}^{(p)}\tilde{B}^{(q)} - \tilde{B}^{(q)}\tilde{B}^{(p)} = O$ $(p, q = 1, 2, \cdots, n)$ is regarded as a system of differential equations for the vector-valued function $\vec{h} = (h_1, \dots, h_n)$. If there is a function $H(x)$ such that $h_i = \partial_{x_{n-i+1}} H$ $(i = 1, \dots, n)$, the system in question is nothing but a WDVV equation for $H(x)$. In this sense, $\tilde{B}^{(p)}\tilde{B}^{(q)} = \tilde{B}^{(q)}\tilde{B}^{(p)}$ $(\forall p,\ q)$ is called a generalised WDVV equation for $\vec{h}$.

From now on we assume that $\vec{h} = (h_1, h_2, \dots, h_n)$ is a solution of a generalised WDVV equation. Then by definition, $\tilde{B}^{(p)}\tilde{B}^{(q)} = \tilde{B}^{(q)}\tilde{B}^{(p)}$ for all p, q. We define a diagonal matrix $B_\infty^{(n)}$ by $B_\infty^{(n)} = \mathrm{diag}(r + w_1, r + w_2, \dots, r + w_n)$ for some constant $r \in \mathbf{C}$. Moreover we define $n \times n$ matrices $B^{(p)}$ $(p = 1, 2, \dots, n)$ by $B^{(p)} = -T^{-1}\tilde{B}^{(p)}B_\infty^{(n)}$ and a system of differential equations

$$\partial_{x_p} Y = B^{(p)} Y \quad (p = 1, 2, \dots, n). \tag{1}$$

Then the system (1) is integrable.

We put $T_0 = x_n I_n - \dfrac{1}{w_n}T$. Since $T - w_n x_n I_n$ does not depend on x_n and since $B^{(n)} = -T^{-1}B_\infty^{(n)}$, the differential equation

$$\partial_{x_n} Y = B^{(n)} Y \tag{2}$$

turns out to be

$$(x_n I_n - T_0)\partial_{x_n} Y = -\frac{1}{w_n} B_\infty^{(n)} Y. \tag{3}$$

Regarding (3) as an ordinary differential equation with respect to the variable x_n, (3) is called an ordinary differential equation of Okubo type. In this sense, the system (1) is one of generalisations of Okubo type ordinary differential equation to several variables case.

As the first application, I explain a relationship between the system of differential equations (1) and algebraic solutions of Painlevé VI. The second application is related with complex reflection groups. If G is an irreducible complex reflection group which is well-generated, there is a polynomial solution of a generalised WDVV equation associated to such a group and the discriminant of G is expressed as the determinant of the matrix EC (which is the matrix obtained from C operated by the Euler vector field E).

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Non-integrability of the Painlevé equations and Stokes phenomenon

In this talk I will present recent non-integrable results for the fourth and fifth Painlevé equations in the sense of the Hamiltonian dynamics. In particular, applying the summability theory to the second variational equations which are linear homogeneous differential equations with an irregular singularity of Pincaré rank 1 at the origin we explicitly compute he corresponding Stokes matrices. These analytic invariants together with the formal invariants generate topologically the differential Galois group. It turns out that the connected component of the unit element of this group is not Abelian. In this way our calculation and Ziglin - Ramis - Morales-Ruiz - Simó method yield the non-integrable results.

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Higher order Painlevé systems, rigid systems and hypergeometric functions

Recently, higher order generalizations of the sixth Painlevé equation (PVI) have been studied from a viewpoint of the monodromy preserving deformations of Fuchsian systems. It is shown in [2, 3] that irreducible Fuchsian systems with a fixed number of accessary parameters can be reduced to finite types of systems by using the Katzs two operations, addition and middle convolution. It is also shown in [1] that the isomonodromy deformation equation is invariant under the Katzs two operations. These facts allow us to construct a classification theory of the isomonodromy deformation equations.

The Fuchsian systems with two or four accessary parameters and four or more singularities are reduced to the systems with the following spectral types ([2, 3]):

The system with the spectral type $\{11; 11; 11; 11\}$ implies PVI as the isomonodromy deformation equations. And the systems with four accessary parameters imply fourth order Painlevé system ([4]). In this talk, we investigate the Fuchsian systems with six accessary parameters and four or more singularities systematically ([5]). They are reduced as follows ([3]):

We also give their particular solutions in terms of the hypergeometric functions which are defined as the solutions of rigid systems ([5]). If time permits, I will discuss a relationship between higher order Painlevé systems and the Appells hypergeometric functions.

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Integrability of certain homogeneous Hamiltonian systems

We investigate a class of natural Hamiltonian systems with two degrees of freedom. The kinetic energy depends on coordinates but the system is homogeneous. Thanks to this property it admits, in general case, a particular solution. Using this solution we derive necessary conditions for the integrability of these system investigating differential Galois group of variational equations.

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Analytic continuation of solutions to nonlinear convolution partial differential equations and its application

In this talk, I will consider the following nonlinear convolution partial differential equation

$$\text{(E)} \qquad \begin{aligned} P(t,x)u =& f(t,x) + \sum_{i+|\alpha|\le m} a_{i,\alpha}(t,x) * \big(\mathscr{M}_{i,\alpha}[\partial_x^\alpha u]\big) \\ &+ \sum_{|\nu|\ge 2} b_\nu(t,x) * \prod_{i+|\alpha|\le m}{}^{*} \big(\mathscr{M}_{i,\alpha}[\partial_x^\alpha u]\big)^{*\nu_{i,\alpha}}, \end{aligned}$$

where $P(t,x)$ is a polynomial of degree l in t with holomorphic coefficients in x, $0 \le l \le m$ and

$$\mathscr{M}_{i,\alpha}[w] = \begin{cases} \dfrac{t^{|\alpha|-1}}{\Gamma(|\alpha|)} * (t^i w), & \text{if } |\alpha| > 0, \\ t^i w, & \text{if } |\alpha| > 0. \end{cases}$$

Under suitable assumptions I will show that if $u(t,x)$ is a holomorphic solution of (E) in a neiborhood of $(0,0) \in \mathbb{C}_t \times \mathbb{C}_x^n$ it can be continued holomorphically to a sector S in t and moreover its extension has some exponential growth order (as $t \longrightarrow \infty$ in the sector).

This result is applied to showing the summability of formal solutions of some nonlinear partial differential equations.

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WKB analysis for the discrete Painlevé equation

As is well-known, each Painlevé equation (except for (PI)) is associated with the Bäcklund transformation, which describes the behavior of solutions under a shift of parameters, and the Bäcklund transformation produces a discrete Painlevé equation. In this talk we would like to consider such discrete Painlevé equations produced through Bäcklund transformations from the viewpoint of the exact WKB analysis. Mainly using (alt-dPI) obtained from the Bäcklund transformation of (PII), we discuss the Stokes geometry and connection formula for such discrete Painlevé equations.

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Asymptotics for the defocusing integrable discrete nonlinear Schrödinger equation

It is well known that the defocusing nonlinear Schrödinger equation $iu_t + u_{xx} - 2|u|^2u = 0$ is integrable. Its integrable discretization

$$i\frac{d}{dt}R_n + (R_{n+1} - 2R_n + R_{n-1}) - |R_n|^2(R_{n+1} + R_{n-1}) = 0$$

was introduced by Ablowitz-Ladik. It has a Lax pair representation and can be solved by using the inverse scattering transform. The solution formula involves an oscillatory Riemann-Hilbert problem with a little complicated phase function. We investigate the asymptotic behavior of the solution $R_n(t)$ as $t \to \infty$ or $|n| \to \infty$. Our tool is the nonlinear steepest descent method of Deift-Zhou.

The asymptotic behavior changes in accordance with the geometry of saddle points of the phase function in the z-plane, where z is the spectral parameter. We consider three regions in the (n, t)-plane. In the first, where $2t > |n|$, there are four saddle points on $|z| = 1$ and the leading part of $R_n(t)$, as $t \to \infty$, is the sum of two terms which decay with oscillation. In the second, near $2t = |n|$, saddle points merge and the leading part is a single term with slower decay. In the third, where $2t < |n|$, the saddle points are off $|z| = 1$ and $R_n(t)$ decays faster than any negative power of n.

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Holonomic system singular along an algebraic curve with cusps

We consider the problem to construct a regular holonomic system singular along a prescribed algebraic curve. The monodromy representation of such system is a representation of the fundamental group of the complement of the curve. It is known that the fundamental group of the complement of an algebraic curve is abelian if the curve is nonsingular, and hence corresponding holonomic system is elementary. In order to get a non-abelian fundamental group, we consider a quartic curve with three cusps, which is known to be the simplest one with such property. For such a curve, we determined all irreducible representations of any rank of the fundamental group. For the case of rank $n = 2$, we can reduce by multiplication the irreducible representation to one where the image of the generators have 1 as their eigenvalues. Then the resulting representation turns out to be the dihedral group D_3. For the cases of rank $n > 2$, there is no irreducible representations. Then we proceed to find a regular holonomic system of rank two with the dihedral group D_3 as monodromy group. Klein showed that any second order Fuchsian ordinary differential equation with finte monodromy group can be obtained as a rational transform of the hypergeometric equation. We used this result to get our holonomic system. As a result, we obtained a regular holonomic system of rank two singular along the quartic curve with three cusps. Note that we needed to add an apparent singularity.

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Borel summability of formal solutions of first order system of PDE

We study the Borel summability of formal solutions of the following equation with respect to a parameter η. More precisely, let $x = (x_1, \ldots, x_n) \in \mathbb{C}^n$, $n \geq 1$ be the variable in $\mathbb{C}^n$. For an integer m with $1 \leq m < n$, let $s_j \in \mathbb{Z}$ $(1 \leq j \leq n)$ be integers such that $s_j \geq 2$ $(j = 1, 2, \ldots, m)$ and $s_j = 1$ for every $j > m$. For $\lambda_j \in \mathbb{C}$, $\lambda_j \neq 0$ $(j = 1, 2, \ldots, n)$ we define

$$\mathcal{L} := \sum_{j=1}^{n} \lambda_j x_j^{s_j} \frac{\partial}{\partial x_j}. \tag{4}$$

Let $N \geq 1$ be an integer and let $f(x, u, \eta) = (f_1(x, u, \eta), \ldots, f_N(x, u, \eta))$ be a holomorphic vector function in a neighborhood of the origin of $x \in \mathbb{C}^n$, where $u = (u_1, \ldots, u_N) \in \mathbb{C}^N$ and $\eta \in \mathbb{C}$ is a complex parameter. We consider the semi-linear system of equations

$$\eta \mathcal{L} u = f(x, u, \eta). \tag{5}$$

We assume

$$f(0, 0, 0) = 0, \qquad \det(\nabla_u f(0, 0, 0)) \neq 0 \tag{6}$$

where $\nabla_u f(0, 0, 0)$ denotes the Jacobi matrix of $f(x, u, \eta)$ with respect to u at the point $x = 0, u = 0, \eta = 0$. In my talk we study the Borel summability of the formal series solution $u = u(x, \eta) = \sum_{\nu=0}^{\infty} \eta^\nu u_\nu(x) = u_0(x) + \eta u_1(x) + \cdots$ with respect to η.

In the case of ordinary differential equations, $n = 1$ the Borel summability of (5) was shown by Balser and Kostov in the regular singular case or by Balser and Mozo-Fernández in the irregular singular case for a linear equation. In our preceeding work with Yamazawa published in FASDE III proceedings we showed the Borel summability in the regular singular case for $n \geq 1$. In the present talk we show the Borel summability in the irregular singular case.

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Linear multi-variable polylogarithms

In my talk I will describe certain class of multi-variable special functions that play analogous role for the Shintani zeta-function as the classical polylogarithm plays for the Riemann zeta-function.

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The center problem for polynomial Abel equation

We prove that, if the Poincaré map $y(0) \longmapsto y(1)$ for solutions $y(x)$ of the polynomial Abel equation $dy/dx = P'(x)y^2 + Q'(x)y^3$ is the identity, then the polynomials P and Q are compositions with a nonconstant polynomial $R(x)$ such that $R(0) = R(1)$.

List of lectures presented on the conference

Takashi Aoki (Kinki University)
The hypergeometric function and WKB solutions

Dennis Bacani (Sophia University)
Lifespan of solutions to nonlinear Cauchy problems with small analytic data

Yuliya Bibilo (IITP RAS)
Inverse monodromy problems and middle convolution

Ewa Ciechanowicz (University of Szczecin)
Value distribution and growth of solutions of the second order ODEs

Peter Clarkson (University of Kent)
Semi-classical orthogonal polynomials and the Painlevé equations

Ellen Dowie (University of Kent)
Rational solutions of the Boussinesq equation and the non-linear Schrödinger equation

Thomas Dreyfus (Université de Toulouse)
Summability of formal solutions of q-difference equations and confluence

Akihito Ebisu (Hokkaido University)
Special values of hypergeometric series

Galina Filipuk (Warsaw University)
On $(q; h)$-Weyl algebras

Renat Gontsov (IITP RAS)
A Maillet type theorem for generalized power series

Yoshishige Haraoka (Kumamoto University)
Connection problem for regular holonomic systems

Stefan Hilger (Katholische Universität Eichstätt-Ingolstadt)
$(q; h)$-deformation of $U(sl(2))$

Sampei Hirose (Shibaura Institute of Technology)
On the relationship between the BNR equation and the Pearcey system

Kunio Ichinobe (Aichi University of Education)
k-summability of formal solutions for certain partial differential equations with time dependent polynomial coefficients

Javier Jimenez-Garrido (University of Valladolid)
Strongly regular sequences, proximate orders and summability

Shingo Kamimoto (Hiroshima University)
Convolutions and analytical continuability of holomorphic functions

Hiroe Kazuki (Josai University)
Ramified irregular singularities of meromorphic connections and plane curve singularities

Hironobu Kimura (Kumamoto University)
Semi classical orthogonal polynomials and general Schlesinger system

Martin Klimeš (Université de Strasbourg)
On the center manifold of unfolded complex saddle-node singularities

Bartosz Langowski (Wrocław University of Technology)
On Sobolev and potential spaces related to Jacobi expansions

Alberto Lastra (University of Alcalá)
Multi-level Gevrey solutions of singularly perturbed linear partial differential equations

Jose Arnie Lope (University of the Philippines Diliman)
Gevrey solutions of a singular integro-differential equation

Grzegorz Łysik (Jan Kochanowski University)
Mean values and real analytic functions

Sławomir Michalik (Cardinal Stefan Wyszyński University)
Summability of divergent solutions of some linear partial differential equations with variable coefficients

Masatake Miyake (Nagoya University)
Exponential growth order of solutions for Moser irreducible system and a counterexample for Barkatou's conjecture

Hiroshi Ogawara (Kumamoto University)
Difference reducibility of linear difference equations

Yasunori Okada (Chiba University)
On coupling equations and their reversibility

Toshio Oshima (Josai University)
Hypergeometric functions with several variables

Bożena Podhajecka (Cardinal Stefan Wyszyński University)
Stokes phenomenon in certain partial differential equations

Javier Sanz (University of Valladolid)
Multi-level asymptotics for singularly perturbed linear partial differential equations with ultraholomorphic coefficients

Jiro Sekiguchi (Tokyo University of Agriculture and Technology)
A generalisation of WDVV equation and flat structures

Andrey Shafarevich (Moscow State University)
Quantization conditions on Riemann surfaces and semiclassical eigenvalues of non-selfadjoint operators

Tsvetana Stoyanova (Sofia University)
Non-integrability of the Painlevé equations and Stokes phenomenon

Takao Suzuki (Kinki University)
Higher order Painlevé systems, rigid systems and hypergeometric functions

Wojciech Szumiński (University of Zielona Góra)
Integrability of certain homogeneous Hamiltonian systems

Hidetoshi Tahara (Sophia University)
Analytic continuation of solutions to nonlinear convolution partial differential equations and its application

Yoshitsugu Takei (RIMS, Kyoto University)
WKB analysis for the discrete Painlevé equation

Hideshi Yamane (Kwansei Gakuin University)
Asymptotic behavior of solutions of the defocusing integrable discrete nonlinear Schrödinger equation

Takuya Yamashiro (Kumamoto University)
Holonomic system singular along an algebraic curve with cusps

Masafumi Yoshino (Hiroshima University)
Borel summability of formal solutions of first order system of PDE

Michał Zakrzewski (Jan Kochanowski University)
Linear multi-variable polylogarithms

Henryk Żołądek (Warsaw University)
The center problem for polynomial Abel equation

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Analytic, Algebraic and Geometric Aspects of Differential Equations

September 14 – 19, 2015

Conference Program

Monday, September 14

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9.00–9.15 OPENING CEREMONY

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() **Chairman: ()**

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Żołąkek

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9.20–10.10 Peter Clarkson (University of Kent)
Semi-classical orthogonal polynomials and the Painlevé equations

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10.10–10.40 Coffee break

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10.40–11.30 Hironobu Kimura (Kumamoto University)
Semi classical orthogonal polynomials and general Schlesinger system

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11.40–12.30 Yoshishige Haraoka (Kumamoto University)
Connection problem for regular holonomic systems

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13.00 Lunch

() () () **Chairman: ()**

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David Guzzetti (SISSA)
TBA

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15.00–15.25 Ewa Ciechanowicz (University of Szczecin)
Value distribution and growth of solutions of the second order ODEs

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15.30–15.55 Takao Suzuki (Kinki University)
Higher order Painlevé systems, rigid systems and hypergeometric functions

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16.00–16.25 Thomas Dreyfus (Université de Toulouse)
Summability of formal solutions of q-difference equations and confluence

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16.25–16.40 Coffee break

() () () **Chairman: ()**

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16.40–17.15 Tsvetana Stoyanova (Sofia University)
Non-integrability of the Painlevé equations and Stokes phenomenon

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17.20–18.45 Wojciech Szumiński (University of Zielona Góra)
Integrability of certain homogeneous Hamiltonian systems

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17.50–18.15 Takuya Yamashiro (Kumamoto University)
Holonomic system singular along an algebraic curve with cusps

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18.20–18.45 Hiroshi Ogawara (Kumamoto University)
Difference reducibility of linear difference equations

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19.00 Dinner

Tuesday, September 15

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() () () **Chairman: ()**
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9.00–9.50 Javier Sanz (University of Valladolid)
Multi-level asymptotics for singularly perturbed linear partial differential equations with ultraholomorphic coefficients
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9.50–10.20 Coffee break
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10.20–11.10 Jiro Sekiguchi (Tokyo University of Agriculture and Technology)
A generalisation of WDVV equation and flat structures
()
11.20–12.10 Toshio Oshima (Josai University)
Hypergeometric functions with several variables
()
12.20–12.45 Andrey Shafarevich (Moscow State University)
Quantization conditions on Riemann surfaces and semiclassical eigenvalues of non-selfadjoint operators
()
12.50 Conference picture
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13.00 Lunch

() () () **Chairman: ()**
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15.00–15.25 Michał Zakrzewski (Jan Kochanowski University)
Linear multi-variable polylogarithms
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15.30–15.55 Akihito Ebisu (Hokkaido University)
Special values of hypergeometric series
()
16.00–16.25 Ellen Dowie (University of Kent)
Rational solutions of the Boussinesq equation and the non-linear Schrödinger equation
()
16.25–16.40 Coffee break

() () () **Chairman: ()**
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16.40–17.05 Renat Gontsov (IITP RAS)
A Maillet type theorem for generalized power series
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17.10–17.35 Yuliya Bibilo (IITP RAS)
Inverse monodromy problems and middle convolution

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17.40–18.05 Sampei Hirose (Shibaura Institute of Technology)
On the relationship between the BNR equation and the Pearcey system

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18.10–18.35 Martin Klimeš (Université de Strasbourg)
On the center manifold of unfolded complex saddle-node singularities

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19.00 Conference Dinner

Wednesday, September 16

() () () **Chairman:** Gavrilov

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9.00–9.50 Shingo Kamimoto (Hiroshima University)
Convolutions and analytical continuability of holomorphic functions

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9.50–10.20 Coffee break

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10.20–11.10 Henryk Żołądek (Warsaw University)
The center problem for polynomial Abel equation

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11.20–12.10 Masafumi Yoshino (Hiroshima University)
Borel summability of formal solutions of first order system of PDE

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12.30 Lunch

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13.15–19.00 Excursion

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19.15 Dinner

Thursday, September 17

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() () () **Chairman: ()**
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9.00–9.50 Takashi Aoki (Kinki University)
The hypergeometric function and WKB solutions
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9.50–10.20 Coffee break
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10.20–11.10 Yoshitsugu Takei (RIMS, Kyoto University)
WKB analysis for the discrete Painlevé equation
()
11.20–12.10 Stefan Hilger (Katholische Universität Eichstätt-Ingolstadt)
$(q;h)$**-deformation of** $U(sl(2))$
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12.20–12.45 Hiroe Kazuki (Josai University)
Ramified irregular singularities of meromorphic connections and plane curve singularities
()
13.00 Lunch

() () () **Chairman: ()**
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15.00–15.25 Dennis Bacani (Sophia University)
Lifespan of solutions to nonlinear Cauchy problems with small analytic data
()
15.30–15.55 Kunio Ichinobe (Aichi University of Education)
k-summability of formal solutions for certain partial differential equations with time dependent polynomial coefficients
()
16.00–16.25 Javier Jimenez-Garrido (University of Valladolid)
Strongly regular sequences, proximate orders and summability
()
16.25–16.40 Coffee break

() () () **Chairman: ()**
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16.40–17.05 Jose Arnie Lope (University of the Philippines Diliman)
Gevrey solutions of a singular integro-differential equation
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17.10–17.35 Yasunori Okada (Chiba University)
On coupling equations and their reversibility
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17.40–18.05 Hideshi Yamane (Kwansei Gakuin University)
Asymptotic behavior of solutions of the defocusing integrable discrete nonlinear Schrödinger equation

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18.10–18.35 11

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19.00 Bonfire

Friday, September 18

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() () () **Chairman: ()**
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9.00–9.50 Alberto Lastra (University of Alcalá)
Multi-level Gevrey solutions of singularly perturbed linear partial differential equations
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9.50–10.20 Coffee break
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10.20–11.10 Masatake Miyake (Nagoya University)
Exponential growth order of solutions for Moser irreducible system and a counterexample for Barkatou's conjecture
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11.20–12.10 Hidetoshi Tahara (Sophia University)
Analytic continuation of solutions to nonlinear convolution partial differential equations and its application
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12.20–12.45 4
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13.00 Lunch

() () () **Chairman: ()**
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15.00–15.25 Bożena Podhajecka (Cardinal Stefan Wyszyński University)
Stokes phenomenon in certain partial differential equations
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15.30–16.05 Sławomir Michalik (Cardinal Stefan Wyszyński University)
Summability of divergent solutions of some linear partial differential equations with variable coefficients
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16.05–16.20 Coffee break

() () () **Chairman: ()**
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16.20–17.00 Grzegorz Łysik (Jan Kochanowski University)
Mean values and real analytic functions
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17.05–17.30 Galina Filipuk (Warsaw University)
On $(q;h)$-Weyl algebras
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17.30–19.00 Walk to the Lake
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19.00 Dinner

Saturday, September 19

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8.00 Departure