# Holonomic systems

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**Abstract**

Holonomic systems are natural extensions of ordinary differential equations, and appear in physics, representation theory, theory of automorphic functions, and so on. The deformation theory of linear differential equations is in a sense a study of holonomic systems. Thus holonomic systems concern various branches of physics and mathematics, and then seem to be substantial objects. There are many similarities between holonomic systems and ordinary differential equations, and also several differences. Recently, in the theory of Fuchsian ordinary differential equations, a big progress is caused by N. M. Katz ("Rigid Local Systems", Princeton Univ. Press, Princeton, NJ, 1996.) and T. Oshima ("Fractional calculus of Weyl algebra and Fuchsian differential equations", MSJ Memoirs, **28**. Mathematical Society of Japan, Tokyo, 2012.). These results can be applied to the study of holonomic systems, and will bring a new development.

In this lecture, we first explain the new understanding of Fuchsian ordinary differential equations given by Katz and Oshima. Fuchsian ordinary differential equations are classified by using the spectral types, and in each class equations are connected by two operations – addition and middle convolution. Analytic properties of solutions are transmitted by these operations. Next we apply these results to the study of regular holonomic systems. We can define the spectral type and the middle convolution similarly as in ODE case. These notions will become powerful tools. Finally we focus on the difference between Fuchsian ordinary differential equations and regular holonomic systems. For the global analysis of regular holonomic systems, geometry of the singular locus plays decisive role. We explain the mechanism, and also show that topology of hypersurfaces works for constructing holonomic systems. Relation to algebraic solutions of deformation equations will also be discussed.

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