SUB-RIEMANNIAN GEOMETRY AND SUB-ELLIPTIC OPERATORS

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1. Abstract

The main aim of my course is to introduce the geometry of smooth manifolds with distinguished smooth subbundle of the tangent bundle, that we will call sub-Riemannian manifolds if additionally a Riemannian metric is given. The typical examples are contact manifolds and the Heisenberg group. Such geometries describes mechanical systems with kinematic constrains, differential equations with controls, quantum systems with magnetic fields, Yang-Mills magnetic fields and have many other applications. It is well known that the geometry of smooth manifold with a Riemannian metric are closely related to the properties of the elliptic operators on that manifolds, think about the close relations of the geometry of the euclidean plane and the Laplace operator or the heat operator. In the same fashion the sub-Riemanian manifolds are intimately related to degenerate elliptic operators, that under some conditions can be called sub-elliptic operators. We will reveal this relation, explaining the main difference between the Riemannian and sub-Riemannian geometries and it influence on the behavior of the differential operators.

2. Contents

- Riemannian geometry and elliptic operators.
- Sub-Riemannian manifolds.
- Hypoelliptic and sub-elliptic operators.

Key words and phrases. Semi-Riemannian manifolds, sub-elliptic operators, hypoelliptic operators, exponential map, Christoffel symbol, extremals, quaternions.

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- Show-Rashevskii and Hörmander theorems.
- Group of diffeomorphisms of the unit circle and geodesic equations.
- Applications of the infinite dimensional sub-Riemannian geometry to the vision theory.

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