Asymptotic analysis and summability of formal power series

Javier Sanz - Valladolid University

Abstract

For many problems (ODEs, PDEs, difference equations, etc.) it makes sense to look for formal power series solutions which, if found, could well be divergent. However, these formal solutions will frequently have an asymptotic meaning, being representations, in a precise sense, of actual, analytic solutions of the corresponding problem. Summability techniques aim at reconstructing such proper solutions from the formal ones. We will present a slight extension of the successful and well-known technique of k-summability in a direction of the complex plane, put forward by J.-P. Ramis and which was the building block for multisummability, a procedure able to sum any formal solution to a system of meromorphic ordinary differential equations at an irregular singular point. The extension concerns the consideration of Carleman ultraholomorphic classes in sectors, more general than the Gevrey classes appearing in Ramis' theory, and which consist of holomorphic functions whose derivatives' growth is governed in terms of a sequence of real numbers, say M. Whenever M is subject to standard conditions, flat functions in the class are constructed on sectors of optimal opening and, resting on the work of W. Balser on moment summability methods, suitable kernels and Laplace and Borel-type transforms are introduced which lead to a tractable concept of M-summability. We will comment on some applications of this tool to the study of the summability properties of formal solutions to some classes of ordinary and partial differential equations.

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