BOREL SUMMABILITY OF FORMAL SOLUTIONS OF FIRST ORDER SYSTEM OF PDE

MASAFUMI YOSHINO (HIROSHIMA UNIVERSITY)

Abstract. We study the Borel summability of formal solutions of the following equation with respect to a parameter η. More precisely, let $x = (x_1, \dots, x_n) \in \mathbb{C}^n$, $n \geq 1$ be the variable in $\mathbb{C}^n$. For an integer m with $1 \leq m < n$, let $s_j \in \mathbb{Z}$ $(1 \leq j \leq n)$ be integers such that $s_j \geq 2$ $(j = 1, 2, \dots, m)$ and $s_j = 1$ for every $j > m$. For $\lambda_j \in \mathbb{C}$, $\lambda_j \neq 0$ $(j = 1, 2, \dots, n)$ we define

$$\mathcal{L} := \sum_{j=1}^{n} \lambda_j x_j^{s_j} \frac{\partial}{\partial x_j}. \tag{0.1}$$

Let $N \geq 1$ be an integer and let $f(x, u, \eta) = (f_1(x, u, \eta), \dots, f_N(x, u, \eta))$ be a holomorphic vector function in a neighborhood of the origin of $x \in \mathbb{C}^n$, where $u = (u_1, \dots, u_N) \in \mathbb{C}^N$ and $\eta \in \mathbb{C}$ is a complex parameter. We consider the semilinear system of equations

$$\eta \mathcal{L} u = f(x, u, \eta). \tag{0.2}$$

We assume

$$f(0, 0, 0) = 0, \qquad \det(\nabla_u f(0, 0, 0)) \neq 0 \tag{0.3}$$

where $\nabla_u f(0, 0, 0)$ denotes the Jacobi matrix of $f(x, u, \eta)$ with respect to u at the point $x = 0, u = 0, \eta = 0$. In my talk we study the Borel summability of the formal series solution $u = u(x, \eta) = \sum_{\nu=0}^{\infty} \eta^{\nu} u_{\nu}(x) = u_0(x) + \eta u_1(x) + \cdots$ with respect to η.

In the case of ordinary differential equations, $n = 1$ the Borel summability of (0.2) was shown by Balser and Kostov in the regular singular case or by Balser and Mozo-Fernández in the irregular singular case for a linear equation. In our preceeding work with Yamazawa published in FASDE III proceedings we showed the Borel summability in the regular singular case for $n \geq 1$. In the present talk we show the Borel summability in the irregular singular case.

Adress: 1-3-1 Kagamiyama, Higashi-Hiroshima, Hiroshima 739-8526, Japan.
E-mail: yoshino@math.sci.hiroshima-u.ac.jp.