Abstracts

Second Brazilian-Polish Topology Workshop

Warsaw: April 29-30, 2015

Paweł Bilski (IM PAN)

Finite CW-complexes as an inverse limit of finite spaces

Abstract: We show that every finite polyhedron (and therefore finite CW-complex) is homotopy equivalent to the inverse limit of an inverse sequence of finite T_0-spaces. The talk is based on papers:

[1] E. Clader, Inverse limits of finite T_0-spaces, Homology, Homotopy Appl. 11 (2009), no. 2, 223-227.

[2] M. J. Thibault, Homotopy theory of combinatorial categories, Ph. D. 2013, University of Chicago.

Thiago de Melo (São Paulo State University, Brazil)

Generalized triple Whitehead product

Abstract: In this talk we introduce the basic concepts on higher order Whitehead product for maps $f_i : \Sigma A_i \to X$, $i = 1, \ldots, r$. Also we present some computations for the triple spherical product to show that $[\eta_4, \eta_4^2, 2\iota_4]$ is trivial.

Jerzy Jezierski (Warsaw University of Life Sciences - SGGW)

Smooth self-maps of Lie groups realizing the least number of periodic points

Abstract: We consider a map $f : M \to M$, where M is a compact connected manifold, $\dim M \geq 3$. We fix a number $n \in \mathbb{N}$. There are two algebraic homotopy Nielsen type invariants satisfying

$$NF_n(f) = \min\{\#\mathrm{Fix}(g^n); g \sim f;\ g \text{ continuous}\}\ ;$$

$$NJD_n(f) = \min\{\#\mathrm{Fix}(g^n); g \sim f;\ g \text{ smooth}\}$$

for $\dim M \geq 3$.

In general $NF_n(f) \ll NJD_n(f)$. Even in the simply-connected case when the growth of $L(f^n)$ is not linear which was observed by Shub and Sullivan in 1974.

We will ask when $NF_n(f) = NJD_n(f)$? Equivalently, in dimension ≥ 3, when the least number of n-periodic point can be obtained by a smooth map? This turns out to be true for all self-maps of tori, nilmanifolds and solvmanifolds. But for each noncommutative Lie group the power map $f(z) = z^k$ is a counter-example.

Here we consider the problem *for which self maps of Lie groups the equality holds for all iterations*. We consider the self-maps of simply-connected Lie groups. We show how to reduce the problem to a graph.

Danuta Kołodziejczyk (Warsaw University of Technology)

Polyhedra with finite depth

Abstract: In 1968 Borsuk posed a problem: *Is it true that every finite polyhedron homotopy dominates only finitely many different homotopy types?*

The answer to this question is negative — we showed in [DK, Fund. Math. 151, 1996] that there exists a finite polyhedron (even 2-dimensional) dominating infinitely many different homotopy types.

By our next results, counterexamples are frequent and can be found even with polycyclic (and poly-Z) fundamental groups. The first one (3-dimensional) finite polyhedron of this kind was presented in [DK, Proc. Amer. Math. Soc, 129, 2001]. At present we can also obtain more 2-dimensional counterexamples, see [DK, Topology Appl. 159, 2012].

On the other hand, every finite polyhedron with virtually-polycyclic fundamental group has finite depth., i. e., there is a bound on the lengths of all descending sequences of different homotopy types $P \geq X_1 \geq X_2 \ldots$ dominated by this polyhedron [DK, Fund. Math. 197, 2007]. One may ask, if there exists a finite polyhedron with infinite depth.

In this talk we will present some classes of 2-dimensional polyhedra with finite depth. We also discuss some strongly related questions which remain unsolved.

Alice Libardi (São Paulo State University, Brazil)

Some Remarks on Cobordism of maps on $\mathbf{Z}_2$-Witt spaces

Abstract: We extend the notion of cobordism groups to the category of pseudomanifolds and normally non-singular maps $f : X \to Y$, it is also given a general definition of Stiefel-Whitney numbers defined on $\mathbb{Z}_2$-Witt spaces using the Wu classes defined by Goresky and Pardon. To compute these numbers we restrict to normally non-singular maps $f : X \to Y$ where X and Y are locally orientable $\mathbb{Z}_2$-Witt spaces and show in several cases that the cobordism class of a map f guarantees that these numbers are zero. As a consequence, if two maps are cobordant, they have the same characteristic numbers, consequently they are cobordism invariants. To conclude we discuss if the results presented here on cobordism of normally non-singular maps between $\mathbb{Z}_2$-Witt spaces can be considered as a generalized homology theory concerning the intersection homology groups.

Thais Monis (São Paulo State University, Brazil)

A relative Borsuk-Ulam theorem for spaces with a free $\mathbb{Z}_2$-action

Abstract: Let (X, A) be a pair of topological spaces, $T : X \to X$ be a free involution and A be a T-invariant subset of X. A question that naturally arises is whether or not all continuous maps $f : X \to R^k$ from X into the euclidean space R^k have a T-coincidence point, that is, a point $x \in X$ such that $f(x) = f(T(x))$. In this talk we will discuss a sufficient condition and some applications.

Piotr Nowak (IM PAN and University of Warsaw)

Higher large scale homology of products of trees

Abstract: I will present a geometric method of killing higher large scale homology of products of trees. This method applies to uniformly finite homology of Block and Weinberger, controlled coarse homology of Nowak and Spakula and to Dranishnikovs almost equivariant homology. As a consequence we obtain vanishing theorems for higher group homology with ℓ_∞ coefficients of lattices in products of trees. This is joint work with Francesca Diana.

Dominika Pawlik (University of Warsaw)

Boundaries of hyperbolic groups and Markov compacta

Abstract: Surfaces with negative curvature are naturally connected to Gromov hyperbolic groups. One way of understanding them is to study their "boudaries". The boundary is a compactification of the group by an almost homogeneous space which admits a natural uniform convergence action. The boundaries are weakly understood; in particular, the list of known group boundaries is very short. I will show that every such boundary may be presented as a Markov compactum which is the inverse limit of a system of finite CW-complexes with some regularity conditions.

Miroslaw Sobolewski (University of Warsaw)

Proximate fixed point property for products, joins and symmetric products of continua

Abstract: Victor Klee introduced proximate fixed point property which is stronger than fixed point property for compacta. We consider relation between this property and operations over continua including products, joins and symmetric products in the sense of Borsuk-Ulam.