Research conference: „Why bases are basic ?”

Abstracts of Talks

Fock-space models for evolution equations coming from quadratic differential operators

Alexandru Aleman (Lund)

We consider evolution equations induced by an important class of quadratic differential operators which arise by the Weyl quantization of quadratic forms. This class of operators has a number of special features which makes their study quite involved. In general:

1) They are far from being selfadjoint,
2) Their eigenvectors form a minimal set with dense span, but not a Riesz basis,
3) The norm of their resolvent grows exponentially towards infinity within certain regions in the complex plane.

The aim of the talk is to present a model for such operators on Fock spaces in several complex variables, which offers a complex analysis perspective and can be used to address a number of questions about the solutions of these evolution equations. The material is based on joint work with J. Viola.

Strong M-bases of exponentials and reproducing kernels in spaces of entire functions

Anton Baranov (St-Petersburg)

Let $\{x_n\}_{n\in N}$ be a complete and minimal system in a separable Hilbert space H, and let $\{y_n\}$ be its biorthogonal system. The system $\{x_n\}$ is said to be hereditarily complete (or a strong M-basis) if for any $x \in H$ we have $x \in \mathrm{Span}\{(x, y_n)x_n\}$. This property may be understood as a very weak form of the spectral synthesis - a reconstruction of a vector x from its (formal) Fourier series $\sum_n (x, y_n)x_n$.

In the talk, a solution of a long-standing problem of hereditary completeness for exponential systems in $L^2(-a, a)$ (equivalently, reproducing kernels of the Paley-Wiener space PW_a^2) will be presented. It turns out that the nonhereditary completeness may occur, though the defect of incompleteness is always at most one.

We also discuss the hereditary completeness for the reproducing kernels in Hilbert spaces of entire functions introduced by L. de Branges. One of our motivations is the relation (via a functional model) between this problem and the spectral synthesis for rank one perturbations of compact selfadjoint operators. We give a complete description of de Branges spaces where nonhereditarily complete systems of reproducing kernels exist in terms of their spectral measures. As a corollary, we obtain a series of striking examples of rank one perturbations of compact selfadjoint operators for which the spectral synthesis fails up to finite- or even infinite-dimensional defect.

The talk is based on joint works with Yurii Belov, Alexander Borichev and Dmitry Yakubovich.

Multiple sampling and interpolation in the Fock space

Alexander Borichev (Marseille)

The problem of the multiple sampling and interpolation in the Fock space with bounded multiplicities was solved by Brekke and Seip in 1993. This problem is still open in the general case and we are going to discuss some partial results obtained in this direction together with Hartmann, Kellay, and Massaneda.

Weak products, Hankel Operators, and Invariant Subspaces

Stephen Richter (Knoxville, Tennessee)

When studying Hardy and Bergman spaces of analytic functions on a region, it is natural to view them as part of the family of H^p- or L^p_a-spaces, and investigate how properties of the functions and operators on these spaces change as the parameter p changes. For reproducing kernel Hilbert spaces like the Dirichlet space of the unit disc or the Drury-Arveson space of the unit ball of $\mathbb{C}^d$ it is unclear what a natural class of related spaces should be. The weak product $\mathcal{H} \odot \mathcal{H}$ and the space of Hankel symbols $\mathcal{X}(\mathcal{H})$ can be associated with a large class of reproducing kernel Hilbert spaces. They may be considered to be the analogs of H^1 and $BMOA$ from the Hardy space theory. In fact, in some generality one shows that $(\mathcal{H} \odot \mathcal{H})^* = \mathcal{X}(\mathcal{H})$, and that Hankel symbols define operators on $\mathcal{H}$ whose null spaces are invariant for all multiplication operators.

For the Dirichlet- and Drury-Arveson spaces one can work out some of the details and draw interesting conclusions. In this talk I will focus on some joint work with my student James Sunkes regarding the Drury-Arveson space.

Riesz Bases and Frames in Signal Analysis

Alexander Ulanovskii (Stavanger)

I give a short introduction into Sampling Theory for signals with disconnected spectrum, prove some recent results and state some open problems.

The non-equivalence between the trigonometric system and the system of functions with pointwise restrictions on values in the uniform and L^1 norms.

Michał Wojciechowski (IM PAN, Warsaw)

We prove that the system of n consecutive trigonometric characters is not equivalent to any system of functions taking limited number of values in every point, in L^1 and sup norms with constants uniform in n. In particular the trigonometric system and Walsh system are not locally equivalent in this norms. In the proof we use the Green - Sanders quantitative version of the Helson idempotent theorem.

Greedy type bases

Przemysław Wojtaszczyk (ICM and IM PAN, Warsaw)

Various greedy type bases we introduced in recent years, largely motivated by non-linear approximation theory. I will try to present basic concepts and results in this area.