Fractal Geometry and Dynamics
Będlewo, October 12-16, 2015

Abstracts of talks

Ledrappier-Young formula for self-affine measures in higher dimensions
Balázs Bárány

We show that self-affine measures satisfy the Ledrappier-Young formula if the corresponding iterated function system (IFS) satisfies the totally dominated splitting condition. This conditions allows us to define global strong stable foliations on self-affine sets. Moreover, if the IFS satisfies the strong open set condition and the corresponding Furstenberg measures have large enough dimension then the dimension of the self-affine measure is equal to its Lyapunov dimension.

Fourier dimension of random images
Fredrik Ekström

The Fourier dimension of a subset of the real numbers measures the fastest polynomial decay rate of the Fourier transform that can be achieved by a measure concentrated on the set. It is well known that the Fourier dimension of a set is always less than or equal to the Hausdorff dimension of the set, and that there are sets for which the inequality is strict. In this talk, I will describe a construction of a random diffeomorphism that can be used to show that every Borel set of real numbers is diffeomorphic to a set that has equal Fourier and Hausdorff dimensions.

Almost everywhere convergence of ergodic series
Ai Hua Fan

I present some recent works on the almost everywhere convergence/divergence of ergodic series, in particular of the Hilbert ergodic transforms, and of dilated series in classical analysis. Actually we find a criterion of convergence for more general function series, using martingale methods. Some part is a joint work with Joerg Schmeling and some other part with Christophe Cuny.

Random affine code tree fractals without Falconer-Sloan condition

Esa Järvenpää

(Joint work with Maarit Järvenpää, Meng Wu and Wen Wu)

We prove that for random affine code tree fractals the affinity dimension is almost surely equal to the unique zero of the pressure function. As a consequence, we show that the Hausdorff, packing and box counting dimensions of such systems are equal to the zero of the pressure. In particular, we do not assume the validity of the Falconer-Sloan condition which has been essential in the previously known results.

Geometry of self-affine sets

Antti Käenmäki

We consider local and global geometrical properties of self-affine sets in the plane. We study tangent sets of truly self-affine sets. If a set in this class satisfies the strong separation condition and projects to a line segment for sufficiently many directions, then for each generic point there exists a rotation O such that all tangent sets at that point are of the form $O((R \times C) \cap B(0,1))$, where C is a closed porous set. We also show that quadratic curves are the only analytic planar curves containing non-trivial self-affine sets. In higher dimensions, we prove that a compact algebraic surface cannot contain non-trivial self-affine sets. This talk is based on recent works with De-Jun Feng, Henna Koivusalo, and Eino Rossi.

Projection theorems for self-affine sets

Thomas Kempton

Over the last couple of years there has been great progress in discussing the set of exceptions for Marstrand's projection theorem applied to self-similar sets. In my talk I will discuss how similar progress can be made in the self-affine case.

On the local Hölder exponents of deRham-like fractal curves

István Kolossváry

(Joint work with Gergely Kiss and Balázs Bárány)

In this talk, we will look at the Hölder exponents of a certain class of fractal curves, which can be considered as a natural generalization of the well-known deRham-curve. These curves are the attractors of Iterated Function Systems (IFS) with affine transformations. We give a characterization of the local Hölder exponents if the linear parts of the corresponding functions are matrices with strictly positive entries. Moreover, we give a condition that the curve is non-differentiable but the Lebesgue-typical Hölder exponent is strictly bigger than 1.

On the exact Hausdorff dimension of the Liouville numbers in the field of formal Laurent series

Poj Lertchoosakul
(Joint work with Radhakrishnan Nair)

We investigate the complexity of the set L of Liouville numbers in the field of formal Laurent series over a finite field. In this setting, we shall discuss some results analogous to those on the real numbers concerning the size and dimension of L. In particular, we show that it has both Haar measure and Hausdorff dimension zero. It is then natural to ask whether L has an exact Hausdorff gauge function or not, and if not, then we should ask for the exact cut-point of the gauge functions. Note that these questions in the classical real case had been open for more than 20 years until they were solved by L. Olsen and D.L. Renfro in 2006. In this setting, we extend the results of Olsen and Renfro by locating the exact cut-point at which the h-dimensional Hausdorff measure of L drops from infinity to zero. Our results also include the fact that if L has infinite h-dimensional Hausdorff measure, then it does not have σ-finite h-dimensional Hausdorff measure.

Multistable and self-stabilizing processes

Jacques Lévy-Véhel

Multistable processes are non-homogeneous extensions of stable processes where the stability index is allowed to vary in time. They provide useful models when the local intensity of jumps if not constant over time, such as is the case for instance for most financial logs or natural terrains. These extensions may be additive processes or not. We will in particular present two multistable extensions of Lévy motions, one being additive while the other is not, and elucidate the links between theses versions. We will also provide the Hausdorf and large deviation multifractal analyses of these processes. Finally, we will present generalizations of multistable processes where the time-dependent stability index deterministically depends on the value of the process at each time. This is again motivated by application in financial analysis.

Diophantine approximation of fractional parts of powers of real numbers

Lingmin Liao
(Joint work with Yann Bugeaud and Michał Rams)

For a real number x larger than 1, the distribution of its powers modulo one is studied. The Hausdorff dimensions of the sets of x, such that the sequence x^n modulo one is close to a given sequence with a given speed, are calculated. Let $\|x\|$ be the distance of x to the nearest integer. Among others, it is proved that for any given real numbers y and b with $b > 1$, the set of points $x > 1$ such that for any big N the inequality $\|x^n - y\| < b^{-N}$ has a solution n between 1 and N, has Hausdorff dimension one.

Unexpected distribution phenomenon resulting from Cantor series expansions

William Mance

We explore in depth the number theoretic and statistical properties of certain sets of numbers arising from their Cantor series expansions. As a direct consequence of our main theorem we deduce numerous new results as well as strengthen known ones.

TBA

Jun Jie Miao

Infinite IFS and related problems

Roman Nikiforov

We establish several new fractal and number theoretical phenomena connected with expansions which are generated by infinite linear iterated function systems. First of all we show that the systems of cylinders of generalized Luroth expansions are, generally speaking, not faithful for the Hausdorff dimension calculation. On the other hand, rather general sufficient conditions for the faithfulness of such covering systems are also found. As a corollary of our main results, we obtain the non-faithfullness of the family of cylinders generated by the classical Luroth expansion. We develop new approach to the study of subsets of essentially non-normal numbers defined by infinite linear IFS and prove that this set is superfractal.

Self-similar sets from the topological point of view

Magdalena Nowak

(Joint work with Taras Banakh and Filip Strobin)

Consider the finite family $\mathcal{F}$ of continuous self-maps on the topological space X and the space $\mathcal{H}(X)$ of nonempty, compact subsets of X. We can define a dynamical system $(\mathcal{H}(X), \mathcal{F})$ such that for each $K \in \mathcal{H}(X) \quad \mathcal{F}(K) = \bigcup_{f\in\mathcal{F}} f(K)$.

By the *self-similar set* or *fractal* we understand a nonempty compact set $A \subset X$ such that

$$A = \mathcal{F}(A) = \bigcup_{f\in\mathcal{F}} f(A)$$

and for every compact set $K \in \mathcal{H}(X)$ the sequence $\left(\mathcal{F}^n(K)\right)_{n=1}^{\infty}$ converges to A in the Vietoris topology on $\mathcal{H}(X)$. In the case when X is a complete, metric space and $\mathcal{F}$ contains Banach contractions, the self-similar set is called an *attractor of iterated function system* $\mathcal{F}$ or *IFS-attractor*.

We deal with the following problem: *Detect compact metric spaces which are IFS-attractors (or weaker types of it), or which are homeomorphic to such attractors.*

A self-similar set $A = \bigcup_{f \in \mathcal{F}} f(A)$, for maps $f\colon A \to A$, is called

- **topological fractal** if A is a Hausdorff space and each $f \in \mathcal{F}$ is *topologically contracting*; for every open cover $\mathcal{U}$ of A there is $n \in \mathbb{N}$ such that for any maps $f_1, \dots, f_n \in \mathcal{F}$ the set $f_1 \circ \dots \circ f_n(A)$ is contained in some set $U \in \mathcal{U}$;
- **Banach fractal** if A is metrizable and homeomorphic to some IFS-attractor;
- Euclidean fractal if A is metrizable and homeomorphic to some IFS-attractor in Euclidean space;
- Banach ultrafractal if A is metrizable, the family $(f(A))_{f \in \mathcal{F}}$ is disjoint and for any $\lambda > 0$ there exist an ultrametric generating the topology of A, such that each $f \in \mathcal{F}$ has $\mathrm{Lip}(f) < \lambda$.

In general the following implications are true

$$\text{Banach ultrafractal} \Rightarrow \text{Euclidean fractal} \Rightarrow \text{Banach fractal} \Rightarrow \text{topological fractal.}$$

We will show that among zero-dimensional spaces all these notions are equivalent. We will present also that each compact set in Euclidean space which contains an open, zero-dimensional, uncountable subset is an Euclidean fractal.

Hausdorff dimension of non-wandering sets of flat spot circle maps

Liviana Palmisano

We study C^2 weakly order preserving circle maps with a flat interval. For the case of the rotation number of bounded type there is a phase transition depending on the degree l of the singularities at the boundary of the flat interval. When $l \leq 2$ the non-wandering set has zero Hausdorff dimension while for $l > 2$ it becomes strictly positive.

For the case of functions with rotation number of unbounded type we expect a much more complicated picture. This is due to to the presence of underlying parabolic phenomena. In the talk I will state a conjecture and present partial results supporting it.

As an illustration of applications I will show how the aforementioned results can be used to study metric and ergodic properties of Cherry flows.

Shrinking targets in parametrised families

Tomas Persson
(Joint work with Magnus Aspenberg)

We consider certain parametrised families of piecewise expanding interval maps, and calculate the Hausdorff dimension of the set of parameters for which the orbit of a point hits a shrinking target infinitely often. The proofs are based on a result by Schnellmann on typicality in parametrised families.

Equidistribution results in negative curvature
Mark Pollicott

In hyperbolic geometry there are a number of classical counting and equidistribution results for lattice points which can be proved using a dynamical viewpoint (e.g., mixing properties of the associated geodesic flow). We will present new variants of these results.

Hausdorff dimension of invariant subsets for certain dynamical systems
Peter Raith

For dynamical systems there is often a relation between the "size" or (better) "thickness" of the space and the "chaoticity" of the system. Usually these relations are in simple cases of the form "Hausdorff dimension equals entropy over Lyapunov exponent". Here this relation is studied first in the one-dimensional case. For invariant subsets R of an expanding piecewise monotonic map $T : [0,1] \to [0,1]$ one obtains that $\dim_H(R) = t_R$, where t_R is the unique zero of $t \mapsto p(R, T, -t \log |T'|)$. If T has constant slope α this gives $\dim_H(R) = \frac{h_{\text{top}}(R,T)}{\alpha}$.
Under certain conditions the Hausdorff dimension behaves stable under small perturbations of the map. From this it can be derived that expanding piecewise monotonic maps can be approximated well by Markov maps.
Next maps $F : [0,1]^2 \to [0,1]^2$ of the form $F(x,y) := \big(Tx, g(x,y)\big)$ are considered. It is assumed that T is a tent map of slope $\alpha \in \big[\sqrt{2}, 2\big]$, and $g(x,y) := \varphi(x) + \lambda\left(y - \frac{1}{2}\right)$ for some $\lambda \in \left(0, \frac{1}{\alpha^2}\right)$ and a linear map $\varphi : [0,1] \to [\frac{\lambda}{2}, 1 - \frac{\lambda}{2}]$. Then $\Lambda := \bigcap_{n=0}^{\infty} F^n\left([0,1]^2\right)$ is the attractor of F. In this case the formula $\dim_H(\Lambda) = 1 + \frac{\log \alpha}{-\log \lambda}$ holds.
To prove this result one has to overcome two problems. The first problem is that T is not a Markov map. Here the above mentioned approximation by Markov maps is used. Using the "transversality condition" one can overcome the second problem, which is that "overlaps" could decrease the Hausdorff dimension.

Everywhere divergence of the one-sided ergodic Hilbert transform
Jörg Schmeling
(Joint work with Ai-Hua Fan and Fredrik Ekström)

For a given number $\alpha \in (0,1)$ and a 1-periodic function f, we study the convergence of the series $\sum_{n=1}^{\infty} \frac{f(x+n\alpha)}{n}$, called one-sided Hilbert transform relative to the rotation $x \mapsto x+\alpha \bmod 1$. Among others, we prove that for any non-polynomial function of class C^2 having Taylor-Fourier series (i.e. Fourier coefficients vanish on $\mathbb{Z}_-$), there exists an irrational number α (actually a residual set of α) such that the series diverges for *all* x. We also prove that for any irrational number α, there exists a continuous function f such that the series diverges for *all* x. The convergence of general series $\sum_{n=1}^{\infty} a_n f(x+n\alpha)$ is also discussed in different cases involving the diophantine property of the number α and the regularity of the function f.

Random sparse sampling in a Gibbs weighted tree
Stéphane Seuret

Consider a Gibbs measure on the dyadic tree, and the associated ń Gibbs ż dyadic tree where each finite word w carries the weight $\mu([w])$. The length of w is denoted by $|w|$. Fix a parameter $\eta \in (0,1)$. We perform a sampling of the Gibbs tree, by keeping each value $\mu([w])$ with probability $2^{-|w|\eta}$, otherwise we replace this value by 0. We study the possibility of reconstructing the initial Gibbs tree from the sampled tree, and perform the multifractal analysis of the remaining structure (which can be viewed as a capacity on the dyadic tree). Various phase transitions occur, both in the reconstruction process and in the multifractal spectra.

Multifractal analysis of some families of random functions motivated by the study of the network traffic
Károly Simon

(Joint work with S. Molnar *(Dept. of Telecommunications and Media Informatics TU Budapest)*, P. Mora *(Morgen Stanley)* and J. Komjathy *(TU Eindhoven)*)

We study the Legendre and Large Deviation multifractal spectra of infinite sums of independent positive random functions $Z(t)$ which can be represented as an infinite sum of random functions $Z_k(t)$ having the following properties:

- $Z_k(t)$ increases in between two consecutive jumps,
- the jumps of $Z_k(t)$ follow a Poisson point process,
- the increments are non-stationary and correlated,
- the graph of $Z_k(t)$ in between two consecutive jumps is determined by a self-affine family of functions.

As a special case our result includes the network traffic generated by the Cubic model of TCP. Our work is a generalization of some of the results of a paper M. Rams and J. L. Vehel (2013).

Hausdorff dimension of limit sets using zeta functions
Polina Vytnova

We study the dependence of the Hausdorff dimension of the limit set of a hyperbolic Fuchsian group on the geometry of the associated Riemannian surface. In particular, we establish the type and location of extrema subject to restriction on the total length of the boundary geodesics. Our approach is based on numerical experiments for which I will give details. I will also state open questions based on numerical results.

Ergodic optimization and prevalence

Yiwei Zhang

Given a dynamical system, we say that a performance function has property P if its time averages along orbits are maximized at a periodic orbit. It is conjectured by several authors that for sufficiently hyperbolic dynamical systems, property P should be typical among sufficiently regular performance functions. In contrast to the literatures by considering typicality in the topological sense, we first address this problem using a probabilistic notion of typicality that is suitable to infinite dimension: the concept of prevalence as introduced by Hunt, Sauer and Yorke. For the one-sided shift on two symbols, we prove that property P is prevalent in spaces of functions with a strong modulus of regularity. Our proof uses Haar wavelets to approximate the ergodic optimization problem by a finite-dimensional one, which can be conveniently restated as a maximum cycle mean problem (a fundamental problem in combinatorial optimization) on a de Bruijin graph.