# Geometric Function Theory meets Geometric Group Theory
# Banach Center IM PAN, Warsaw

October 14-17, 2015

# Abstracts

## October 12, 2015

**Jonas Azzam**, Universitat Autònoma de Barcelona, Spain

Title: *A characterization of rectifiable measures*

Abstract: The analyst's traveling salesman theorem of Peter Jones gives a necessary and sufficient condition for when one can construct a curve that contains *all* of a given set. The condition is given in terms of $\beta$-numbers, which measure the flatness of the set at various scales and locations. In this talk, we will survey some old and recent results that deal with the problem of classifying when one can construct curves (or surfaces) that trace out *most* of a set, where by *most* we mean up to a set of measure zero with respect to some specified measure. In particular, using a variant of Jones' original $\beta$-numbers, we have a characterization of when this happens assuming the given measure is absolutely continuous with respect to Hausdorff measure. This is joint work with Xavier Tolsa.

**Marc Bourdon**, Université Lille, France

Title: *Hyperbolic spaces and conformal dimension*

Abstract: The conformal dimension of the boundary of a (Gromov) hyperbolic space is a quasi-isometry invariant of the space. It is bounded by below by the topological dimension of the boundary, and by above by its Hausdorff dimension. The conformal dimension will be presented, and we will be interested in determining the hyperbolic groups for which the conformal dimension of the boundary is equal to the topological dimension.

**Anna Kaźmierczak** and **Antoni Pierzchalski**, University of Łódź, Poland

Title: *Some remarks on modules of foliations*

Abstract: We study modules of orthogonal foliations of a Riemannian manifold. A pair of orthogonal foliations (of complementary dimensions) on $\mathbb{R}^3$ or $\mathbb{R}^2$ has a physical interpretation. One (1-dimensional) foliation corresponds to the lines of an electric field, whereas the leaves of the other one are the level sets of the potential of this field. We specify sufficient conditions under which the product of modules of any finite number of orthogonal foliations is equal to 1. We formulate two theorems. The conditions in the first one are expressed in the language of conjugate submersions, whereas assumptions of the other one refer to the behavior of the Jacobian along the leaves.

**Juhani Koivisto**, University of Helsinki, Finland

Title: *Hyperbolic spaces, amenability, and Sobolev inequalities*

Abstract: An infinite discrete hyperbolic group is amenable if and only if it is virtually cyclic, in which case its boundary contains two points. Here, we consider the class of uniformly coarsely proper visual Gromov hyperbolic metric spaces. Given such a space $X$, we show that if its boundary is connected and uncountable, its fundamental class in the controlled coarse homology vanishes, and $X$ is non-amenable. In particular, we obtain a metric proof of the fact that any locally compact compactly generated non-elementary hyperbolic group with connected boundary is not geometrically amenable. Finally, we consider the class of quasiconvex uniform metric measure spaces supporting a local weak $(1,1)$-Poincaré inequality. Given such a space, we show that it is non-amenable (more generally, its fundamental class in the controlled coarse homology vanishes) if and only if it satisfies a (weighted) $(1,1)$-Sobolev inequality. Previously, this has been proved by P. Nowak and J. Spakula for discrete metric spaces with counting measure.

**Tomasz Kostrzewa**, Warsaw Institute of Technology, Poland

Title: *Sobolev spaces, metric spaces and LCA groups*

Abstract: In recent years different definitions of Sobolev spaces on metric measure spaces were intensively studied. P. Górka and E.G. Reyes introduced Sobolev spaces on general LCA groups and studied they imbedding properties [1]. This brings up the question *"What happens if we have both structures combined: that of a metric measure space and of a LCA group?"* In this talk we will try to show some results which are related to this problem. This talk is based on joint work with P. Górka [2, 3, 4].

Bibliography:

1 P. Górka, E. G. Reyes, Sobolev spaces on locally compact abelian groups and the bosonic string equation. J. Aust. Math. Soc. to appear.

2 P. Górka, T. Kostrzewa, E. G. Reyes, The Rellich lemma on compact abelian groups and equations of infinite order. Int. J. Geom. Methods Mod. Phys. 10 (2013), no. 2, ID 1220030, 11p.

3 P. Górka, T. Kostrzewa, E. G. Reyes, Sobolev spaces on locally compact abelian groups: compact embeddings and local spaces. J. Funct. Spaces 2014, Art. ID 404738, 6pp.

4 P. Górka, T. Kostrzewa, Sobolev spaces on metrizable groups. Ann. Acad. Sci. Fenn. Math. 40 (2015), 837-849.

**Urs Lang**, ETH Zurich, Switzerland

Title: *Structure of injective hulls of graph metrics*

Abstract: We are interested in Isbell's injective hull for integer valued metrics corresponding to connected infinite graphs. Under suitable assumptions, the injective hull is proper and has the structure of a locally finite polyhedral complex with finitely many isometry types of cells in every dimension. This provides geometric models for certain finitely generated groups, word hyperbolic groups in particular, with some weak properties of nonpositive curvature. The talk will survey a number of results on the geometric and combinatorial structure of these polyhedral complexes and will conclude with some open questions.

**Rami Luisto**, University of Helsinki, Finland

Title: *A characterization result for BLD-mappings and applications*

Abstract: We discuss a theorem characterizing BLD-mappings as discrete Lipschitz quotient mappings in a metric setting. As an application we obtain a new proof for a limit theorem of BLD-mappings. Martio and Väisälä showed in 1988 that between Euclidean domains the limit of $L$-BLD-mappings is $L$-BLD. This was generalized by Heinonen and Rickman to show that between generalized manifolds of type A the limits of $L$-BLD-mappings are $K$-BLD with a quantitative constant $K$. With the new characterization result we can show that in fact $K = L$.

**John MacKay**, University of Bristol, UK

Title: *Boundaries of relatively hyperbolic groups*

Abstract: A key invariant of a hyperbolic group is its boundary at infinity, which is a metric space canonically defined up to quasisymmetry. In this talk I will discuss on-going work with Alessandro Sisto looking at the metric structure of the boundary at infinity of relatively hyperbolic groups.

**Damian Osajda**, University of Wrocław, Poland

Title: *Embedding expanders in groups and applications*

Abstract: I will present my recent construction of finitely generated groups containing isometrically embedded expanders. Such groups have many exotic properties: they

do not embed coarsely into Hilbert space, the Baum-Connes conjecture with coefficients fails for them. The construction allows to provide the first examples of groups that have no property $A$ (are not exact) but still are coarsely embeddable into a Hilbert space. Even more: they act properly on $CAT(0)$ cubical complexes. I will present some further applications of the main construction concerning aspherical manifolds and the asymptotic dimension.

**Pekka Pankka**, University of Jyväskylä, Finland

Title: *Quasiregular maps into closed manifolds*

Abstract: Quasiregular mappings have the same role in geometric function theory as the holomorphic maps in the classical function theory of one complex variable. From the point of view of GFT, one of the main questions related to quasiregular mappings is which manifolds receive a quasiregular map from the Euclidean $n$-space. For mappings into closed manifolds, this leads to questions on the fundamental group of the target. In this talk I will discuss a result with Rami Luisto that maximal growth of the fundamental group in the target implies that the group is virtually abelian, and a related result with Enrico Le Donne that, if we restrict to so-called BLD-mappings, the target always has a virtually abelian fundamental group.

**Manuel Ritore**, Universidad de Granada, Spain

Title: *Recent regularity results for surfaces of prescribed mean curvature in the first Heisenberg group*

Abstract: When treating geometric inequalities involving the sub-Riemannian area by variational methods, one of the main open problems is the optimal regularity of the minimizers in the Heisenberg groups. We shall describe recent advances on this problem as well as a characterization of minimizers with weak prescribed mean curvature recently obtained in a joint work with Matteo Galli. Time permits, we shall also treat the Bernstein result for $C^1$ minimizers recently obtained by Galli and the speaker.

**Elefterios Soultanis**, University of Helsinki, Finland

Title: *Existence of p-energy minimizers in homotopy classes of Newtonian maps between metric measure spaces*

Abstract: The talk ties together aspects of GFT and GGT by considering maps minimizing an energy functional in a given homotopy class of maps. The context is that of Newtonian maps from a compact Poincaré space to a compact metric space of nonpositive curvature (NPC-space).

I introduce a notion of homotopy between Newtonian maps and discuss how methods from geometric group theory are used to prove the existence of minimizers under the assumption that the target space has hyperbolic fundamental group (or equivalently, hyperbolic universal cover.

**Ben Warhurst**, University of Warsaw, Poland

Title: *Prime ends in the Hesenberg group*

Abstract: This talk will discuss aspects of a theory of prime ends in the Heisenberg group. In particular we will discuss an equivalent theory to that developed by Näkki in the Euclidean setting and its application to boundary extension of quasiconformal mappings.

The talk is based on the joint work with Tomasz Adamowicz and the research has been conducted within the Grant Iuventus Plus.

**Aleksandra Zapadinskaya**, Universita di Pisa, Italy

Title: *Gromov's dimension comparison estimate for rectifiable sets in stratified groups*

Abstract: Gromov established that the Hausdorff dimension of $(q-1)$-dimensional topological submanifolds of an equiregular Carnot-Carathéodory space cannot be smaller than $Q-1$, where $Q$ is the Hausdorff dimension of the equiregular Carnot-Carathéodory space. Using a topological argument, he reduced the proof of this fact to the case of smooth hypersurfaces, for which this dimensional estimate is a consequence of the transverality of their tangent spaces with respect to the horizontal distribution. Thus, it is reasonable to ask whether an analogous dimensional estimate is true for $(q-1)$-rectifiable sets, for which the notion of tangent space is rather natural. Balogh and Tyson showed that in this case one can find suitable rectifiable sets of $BV$ regularity, violating the dimensional estimate. Positive results for Sobolev surfaces in the Heisenberg group have been obtained by Balogh, Tyson, Wildrick and Magnani, Malý, Mongodi. In a joint work with V. Magnani, we generalize these results to Carnot groups. Our main tool is a suitable weak exterior differentiation for pull-back differential forms that is weaker than the classical distributional differentiation.