III Meeting on Lie systems

PROGRAM

Monday, September 21

10:00 J.F. Cariñena Marzo (A generalization of Lie Theorem of integrability by quadratures)
11:00 Coffee
11:30 V. Salnikov (Graded geometry in gauge theories: above and beyond)

Tuesday, September 22

10:00 J. de Lucas (Geometry and applications of Lie–Hamilton systems on the plane)
11:00 Coffee
11:30 F.J. Herranz (The Cayley–Klein approach to symmetrical homogenous spaces: isometries, trigonometry and conformal symmetries)
12:30 M. Tobolski (Lie–Hamilton systems on curved spaces)

Wednesday, September 23

10:00 J. Gutt (Wei–Norman equations and cominuscule induction)
11:00 Coffee
12:00 C. Sardón (Quantum Lie systems: two-examples)

Thursday, September 24

10:00 J. Clemente–Gallardo (Kähler-Lie systems on the space of states I: geometry)
11:00 Coffee
11:30 J.A. Jover Galtier (Kähler-Lie systems on the space of states II: applications)

Friday, September 25

10:00 Discussion about the future research and cooperation
11:00 Coffee
11:30 Summing up and closing

Talks and coffee breaks take place in rooms 403 and 409 (4 floor), respectively.

ABSTRACTS

Monday, September 21

A generalization of Lie Theorem of integrability by quadratures

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Abstract

The classical results of Lie on integrability by quadratures will be reviewed and some generalizations will be proposed. After a short review of the classical Lie theorem, a finite dimensional Lie algebra of vector fields is considered and the most general conditions under which the integral curves of one of the fields can be obtained by quadratures in a prescribed way will be discussed, determining also the number of quadratures needed to integrate the system. The theory will be illustrated with examples and an extension of the theorem where the Lie algebras are replaced by some distributions will also be presented.

References

[1] R. Campoamor-Stursberg, J.F. Cariñena and M.F. Rañada, *Higher-order superintegrability of a Holt related potential*, J. Phys. A: Math. Theor. **46**, 435202 (2013).

[2] J.F. Cariñena, M. Falceto, J. Grabowski and M.F. Rañada, *Generalized Lie approach to integrability by quadratures*, J. Phys. A: Math. Theor. **48**, 215206 (2015).

[3] C.R. Holt, *Construction of new integrable Hamiltonians in two degrees of freedom*, J. Math. Phys. **23**, 1037–1046 (1982).

[4] V.V. Kozlov, *Integrability and nonintegrability in Hamiltonian mechanics*, Russian Math. Surveys **38**, 1–76 (1983).

[5] V.V. Kozlov, *The Euler-Jacobi-Lie integrability theorem*, Regul. Chaotic Dyn. **18**, 329–343 (2013).

[6] A. Mishchenko and A. Fomenko, *Generalized Liouville method of integration of Hamiltonian systems*, Funct. Anal. Appl. **12**, 113–121 (1978).

Graded geometry in gauge theories: above and beyond

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Abstract

We study some graded geometric constructions appearing naturally in the context of gauge theories. We introduce the language of Q-bundles convenient for description of symmetries of sigma models. Inspired by a known relation of gauging with equivariant cohomology we generalize the latter notion to the case of arbitrary Q-manifolds introducing thus the concept of equivariant Q-cohomology. It turns out to be useful for analysis of such theories as the (twisted) Poisson sigma model and the Dirac sigma model. We obtain these models by a gauging-type procedure of the action of a group related to Lie algebroids and n-plectic manifolds. We also show that the Dirac sigma model is universal in space-time dimension 2. This is a joint work with Thomas Strobl, and in part with Alexei Kotov.

Tuesday, September 22

Geometry and applications of Lie–Hamilton systems on the plane

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Abstract

A Lie–Hamilton system is a nonautonomous system of first-order ordinary differential equations describing the integral curves of a t-dependent vector field taking values in a finite-dimensional real Lie algebra of Hamiltonian vector fields with respect to a Poisson structure [1]. We provide new algebraic/geometric techniques to easily determine the properties of such Lie algebras on the plane, e.g., their associated Poisson bivectors. We study new and known Lie–Hamilton systems on $\mathbb{R}^2$ of physical, biological and mathematical interest [2]. New results cover Cayley-Klein Riccati equations, the here defined planar diffusion Riccati systems, complex Bernoulli differential equations and projective Schrödinger equations. Constants of motion for planar Lie–Hamilton systems are explicitly obtained which, in turn, allow us to derive superposition rules through a coalgebra approach [3].

References

[1] J.F. Cariñena, J. de Lucas and C. Sardón, *Lie–Hamilton systems: theory and applications*, Int. J. Geom. Methods Mod. Phys. **10**, 09129823 (2013).

[2] A. Blasco, F.J. Herranz, J. de Lucas and C. Sardón, *Lie–Hamilton systems on the plane: applications and superposition rules*, J. Phys. A **48**, 345202 (2015).

[3] A. Ballesteros, J.F. Cariñena, F.J. Herranz, J. de Lucas and C. Sardón, *From constants of motion to superposition rules for Lie–Hamilton systems*, J. Phys. A **46**, 285203 (2013).

The Cayley–Klein approach to symmetrical homogenous spaces: isometries, trigonometry and conformal symmetries

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Abstract

The family of orthogonal Cayley-Klein Lie algebras is revisited. This includes simple and non-simple real Lie algebras which can be obtained by contracting the pseudo-orthogonal algebras $so(p,q)$ of the Cartan series B_l and D_l. Some associated symmetrical homogeneous spaces of constant curvature are next constructed. A given Lie algebra contraction can be interpreted geometrically as the zero-curvature limit of some underlying homogeneous in such a manner that each (graded) contraction parameter is identified with the sectional curvature of a space. The Cayley-Klein framework allows one to study in a unified setting Riemannian spaces (spherical, Euclidean, hyperbolic), pseudo-Riemannian spaces (the relativistic (anti-)de Sitter and Minkowskian spacetimes) as well as semi-Riemannian spaces (the non-relativistic Newtonian and Galilean spacetimes). These results are illustrated by considering the two-dimensional Cayley-Klein spaces (obtained from $so(3)$, $so(2,1)$ and their contractions) which are described in detail by describing vector fields of isometries, trigonometric relations and vector fields of conformal transformations. We stress that these results can further be applied in the construction of Lie and Lie-Hamilton systems on curved spaces.

Lie–Hamilton systems on the curved spaces

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Abstract

The classification of Lie–Hamilton systems on the plane (up to generic points) as well as techniques for obtaining superposition rules for such systems is given in the two papers [1,2]. Among others, above classification presents the system P_1, which we here generalize to the 2-dimensional curved spacetimes, the so-called Cayley-Klein spaces. We give superposition rules for these new Lie-Hamilton systems using unified trigonometry formulas from [3].

References

[1] A. Ballesteros, A. Blasco, F.J. Herranz and C. Sardón, *Lie–Hamilton systems on the plane: properties, classification and applications*, J. Differential Equations **258**, 2873–2907 (2015).

[2] A. Ballesteros, A. Blasco, F.J. Herranz and J. de Lucas, *Lie–Hamilton systems on the plane: applications and superposition rules*, J. Phys. A **48**, 345202 (2013).

[3] F.J. Herranz, R. Ortega and M. Santander, *Trigonometry of spacetimes: a new self-dual approach to a curvature/signature (in)dependent trigonometry*, J. Phys. A **33**, 4525–4551 (2000).

Wednesday, September 23

Graded geometry in gauge theories: above and beyond

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Abstract

We show how the method of so-called 'cominuscule induction' can be used to reduce the equations of Lie type for certain semi-simple Lie groups to a system of Riccati equations.

Quasi–Lie schemes: two examples

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Abstract

A Quantum Lie system is a system described by a t-dependent Hamiltonian which can be split as a linear combination of t-dependent functions and skew-self adjoint operators spanning a finite-dimensional real Lie algebra of operators. In this work we pay attention to a certain generalization of a Quantum Lie system admitting a group of unitary t-dependent transformations. These systems, the so called *Quantum Quasi–Lie systems* (QQLS) are the quantum analog of the classical Quasi–Lie theory and many of the properties retrieved are a mere generalization of the classical theory to the quantum framework. We show two examples of Quantum Quasi–Lie systems: one is the quantum version of the classical nonlinear oscillator formerly studied by Perelemov, the other is an application to Fluid Dynamics equations.

Thursday, September 24

Kähler-Lie systems on the space of states I: geometry

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Abstract

The aim of this talk is to introduce the main geometrical tools required for the second part. We shall summarize the main properties of the geometrical description of the space of states of a quantum system, focusing on the relations with the Schrödinger and Heisenberg formalisms.

Kähler-Lie systems on the space of states II: applications

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Abstract

We employ the techniques of the previous talk to study several types of Lie systems appearing in Quantum Mechanics and their superposition rules. Our results are applied in the analysis of qubit systems.