Nonlinear Control and Geometry

Banach Center Conference, Będlewo, 24-28 August 2015

ABSTRACTS

John Baillieul (Boston University)

Title: Topological aspects of optimal information acquisition in robotic exploration and multimodal sensor fusion.

Abstract: The talk presents some recent work to construct mathematical models of the information content of spatially varying scalar fields defined on R^1,R^2, and R^3. The models are developed using analytical tools from both differential topology and information theory. The concept of topological persistence is described, and a refined notion that we refer to as topological information utility is presented. We describe applications in which the information content of measurements of continuously varying scalar fields can be used to guide robotic exploration or to support a new approach to multi-sensor data fusion with the aim of creating a composite image of a geometric locale that maximizes the information available to an observer. Because the model rests on information theory, it provides the foundation for treating search as a problem in optimal information acquisition. Because there is also a connection to topology and geometry, search problems may be thought of in terms of climbing information gradients.

Maria Barbero-Linan (Universidad Carlos III de Madrid-ICMAT)

Title: Geometric description of controllability of hybrid control systems.

Abstract: Hybrid systems with or without controls are those systems where there is an interaction between continuous dynamics and discrete events. Some examples of hybrid systems are given by a bouncing ball, an automobile with automatic or manual transmission, impacts, thermostats, etc. These systems attract the attention of engineers, computer scientists and mathematicians.

Recently, mathematicians have focused on the geometrization of hybrid systems in order to bring more understanding to all the possible case studies [2,3,5]. A hybrid control system can be understood as a family of generalized dynamical systems with interactions among them. That information can be summarized in a directed graph. These interactions make necessary a casuistic approach to study hybrid control systems. The notion of controllability remains the same as in classical control theory, that is, a hybrid control system is controllable if for any two points there exists an admissible trajectory that joins them [1,4]. However, for hybrid systems, not only the involutive distribution of the control vector fields contributes to the reachable points, but also the set of points where jumps between systems take place. That is why single control systems could be no controllable, but interactions among them could make the hybrid control system controllable. In this talk we first define geometrically hybrid control systems, then focus on controllability tests for such systems. Some examples will be provided to elucidate the importance and necessary properties of the jump set to guarantee controllability.

References:

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Mohamed Ali Belabbas (University of Illinois)
Title: Geometry of optimal sensor/actuator placement.

Abstract: Observers for linear systems are ubiquitous, with applications in fields ranging from engineering to biology to economics. The Kalman filter is known to be the optimal estimator of the state when the noise is additive and Gaussian. Because its performance is limited by the sensors to which it is paired, it is natural to seek an optimal sensor for the Kalman filter. The problem is however not convex and, as a consequence, many ad hoc methods have been used over the years to design sensors. We show how to characterize and obtain the optimal sensor for the Kalman filter. Precisely, we exhibit a positive definite operator which optimal sensors have to commute with. We furthermore provide a gradient flow to find optimal sensors, and prove the convergence of this gradient flow to the unique minimum in a broad range of applications. This optimal sensor yields the lowest possible estimation error for measurements with a fixed signal to noise ratio. The results presented here also apply to the dual problem of optimal actuator design.

Bernard Bonnard (Université de Bourgogne, Dijon)
Title: Geometric optimal control: The Copepod swimmer vs Purcell three links swimmer.

Abstract: A seminal contribution about micro animal swimming was proposed by Purcell and is known as the three links Purcell swimmer. This leads to a complicated 5-dimensional model. In this talk we discuss a simplified system modelling the copepod swimming which allows a geometric analysis in the framework of SR-geometry and open the road to practical experiments based on geometric optimal control.

Ugo Boscain (Ecole Polytechnique-CNRS, Paris)
Title: Geodesics, Laplacians and random walks in sub-Riemannian geometries.

Abstract: In this talk I will connect via the random walks point of view, the geodesics and the Laplacian in a sub-Riemannian manifold M. This problem is not trivial even in the Riemannian context and passes through the definition of a volume. We define two type of Laplacians: the macroscopic one as the divergence of the horizontal gradient once a volume in the ambient space is fixed and the microscopic one as the operator associated with a geodesic random walk. This second definition requires to fix a probability density on the cylinder of initial conditions of covectors in T^*M_q, where q is the starting point. This cylinder parametrizes the different geodesics. We study under which conditions these two operator coincide. We will see that the result strongly depend on the type of sub-Riemannian structure.

Roger Brockett (Harvard University)
Title: Infimizing Sequences of Sprials and the Optimization of First Bracket Controllable Systems.

Abstract: There are many examples of optimal control problems whose open loop solution requires an infinite number of switches in a finite interval. Of these, perhaps the Fuller problem is the best known and that in that case that there are various senses in which an optimal solution exists. In this talk I will discuss a vaguely analogous optimal control problem for which the infimizing sequences involve an unbounded number of switches and bracketing but no limiting optimal control exists.

Thomas Chambrion (Université de Lorraine)

Title: Averaging methods for the control of closed quantum systems.

Abstract: The evolution of closed quantum systems excited by an external time-variable electric field can usually be mathematically modeled with bilinear PDEs. Many difficulties arise when one wants to study the controllability properties or design explicit control laws for such systems. In particular, the state space is infinite dimensional and the regularity of the operators is not clear from the modeling. PDEs techniques have provided some deep and difficult results but appears to be inefficient for most of the practical applications. This talk will review some of the major results obtained in the last decade by using (finite dimensional) geometric methods for the effective control of infinite dimensional closed quantum systems, with a special focus on some recent positive results in the case of mixed spectrum systems. The main ingredient is an extension of the classical averaging techniques to irregular infinite dimensional operators.

Domenico d'Alessandro (Iowa State University)

Title: Mathematical methods and problems in the control of quantum mechanical systems.

Abstract: In the last decades, technological advances in pulse shaping techniques have opened up the possibility of manipulation of systems whose evolution follows the laws of quantum mechanics. On the other hand, novel applications, such as in quantum information processing, have offered further motivation for this study. From a mathematical point of view, the field which is now known as 'Quantum Control' is a combination of different mathematical techniques borrowed from a wide variety of mathematics areas. Different mathematical tools apply to different models which correspond to different approximations of the physical system. The simplest case is the one of a closed system, i.e., a system non interacting with the environment in any way other than through the external control, whose state can be modeled as a vector in a finite dimensional Hilbert space, and is controlled in open loop. In this case, the operator describing the evolution belongs to a Lie group and the control system is determined by a family of right invariant vector fields on such Lie group. Techniques of geometric control are therefore appropriate in this case. As some of the above assumptions on the physical model are relaxed, different tools have to be used. The consideration of 'open' systems, which also allow for a continuous measurement of the state and feedback, requires the introduction of techniques of dynamical semigroups as well as stochastic calculus. The study of infinite dimensional quantum control systems is often done using tools of functional analysis and control of partial differential equations.

In this talk I will survey the field from the point of view of the mathematics that is used and needs to be developed for the control of quantum systems. After introducing basic notions of quantum mechanics and the relevant models used in applications I will indicate a number of open mathematical problems.

David Martin de Diego (CSIC, Madrid)

Title: Geometric integration for optimal control problems.

Abstract: In this talk we will explore different approaches to the geometric integration of optimal trajectories of mechanical systems. The main idea is to develop, from a geometric point of view, numerical methods for optimal control of Lagrangian mechanical systems. We will use the theory of discrete mechanics and variational integrators (Marsden and West 2001) to derive both an integrator for the dynamics and an optimal control algorithm in a unified manner. This is accomplished through the discretization of the Lagrange-d'Alembert variational principle on manifolds. Integrators for the mechanics are derived using a standard Lagrangian function and virtual work done by control forces, while control optimality conditions are derived using a special Lagrangian defined on a higher dimensional space which encodes the dynamics and a desired cost function. The resulting integration and optimization schemes are symplectic and respect the state-space structure and momentum evolution. These qualities are associated with favorable numerical properties which motivate the development of practical algorithms that can be applied to robotic or aerospace vehicles.

References:

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[3] Kobilarov, M., Marsden, J.E.: Discrete geometric optimal control on Lie groups. IEEE Trans. Robot. 27(4), 641–655 (2011)

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R. El Assoudi-Baikari (INSA Rouen)

Title: Two-step nilpotent sub-Riemannian Lie algebras in dimension 5 and 6.

Abstract: Let $(G, \mathcal{D}, B)$ be a sub-Riemannian Lie group, where G is a real connected and simply connected n-dimensional Lie group, $\mathcal{D}$ is a right-invariant analytic distribution of constant rank $k \leq n-1$ and B is a right-invariant symmetric positive definite bilinear form defined on $\mathcal{D}$. We are interested in the case where G is a 2-step nilpotent Lie group.
The sub-Riemannian geometry on nilpotent Lie groups, whose applications are numerous, has attracted interest of many authors. The basic is the Heisenberg Lie group. Systems on Lie groups are widely used in control theory (for example, nilpotent approximation of control systems). To $(G, \mathcal{D}, B)$, we associate a sub-Riemannian Lie algebra $(\mathfrak{g}, \mathfrak{p}, \mathcal{B})$, by taking $\mathfrak{g} = T_eG$, the Lie algebra of G, $\mathfrak{p}$ a linear subspace of $\mathfrak{g}$ given by $\mathfrak{p} = \mathcal{D}_e$ and $\mathcal{B} = B_e$ is the symmetric positive definite bilinear form defined on $\mathfrak{p}$.
In this talk, we start by presenting a classification of (oriented) sub-Riemannian Lie algebras of dimension n for which the derived Lie algebra $\mathfrak{D}(\mathfrak{g})$ is of dimension one, that contains the contact case. Then we give a complete classification of the 2-step nilpotent sub-Riemannian Lie algebras of dimension $n \leq 6$.
The presented classification allows to study the integrability of the associated geodesic equations, (more precisely, the adjoint equation on the Lie algebra $\mathfrak{g}$).
This is a joint work with W. Respondek and K. Dahamna.

Willem Esterhuizen (MINES Paris Tech)

Title: On barriers in nonlinear control systems with mixed constraints.

Abstract: Given a constrained nonlinear system, the admissible set is a subset of the state space which consists of all the initial conditions for which there exists a control such that the constraints are satisfied for all time. This set is closed and its boundary can be divided into two complementary parts, one of which is called the barrier that exhibits a semi-permeability property: if the state, initiating from the interior of the admissible set, crosses the barrier, then it is guaranteed that it will violate the constraints in the future. Moreover, if the state starts outside the admissible set, no admissible trajectory can penetrate the barrier in the direction of the interior of the admissible set. The barrier is proved to satisfy a minimum-like principle which

enables its direct construction. The ideas of barriers and semi-permeability were originally studied by Isaacs in the context of capturability in pursuit-evasion differential games, and was recently introduced to constrained nonlinear control systems, without any optimality requirement, by DeDona and Lévine in SIAM J. Control and Optimiz., Vol. 51, 2013, under the assumption that the state and input constraints were separate. In the current presentation we generalise the previous work to the case where the constraints are mixed, i.e. explicitly depend on the state and the control. Constraints of this type may be encountered in optimal control, and in particular in certain aerospace applications, as well as in constrained flat systems. Under the assumption that the control functions are measurable the evolution of the mixed constraints with respect to time may be discontinuous and this makes the analysis considerably more difficult. However, we show that the barrier and the concept of semi-permeability carry over to the mixed constraint case and that we can construct the barrier via a modified minimum-like principle involving Karush-Kühn-Tucker multipliers and a nonsmooth gradient condition at its endpoints.

Marek Grochowski (Cardinal Wyszyński University, Warsaw)
Title: Reachable sets for sub-Lorentzian structures and associated control-affine systems.

Abstract: I will explain the connection between sub-Lorentzian structures and control-affine systems. Then I will show how the sub-Lorentzian geometry methods can be used to compute reachable sets for control-affine systems with a scalar input.

Dimitry Gromov (St.-Petersburg State University)
Title: On the geometric structure of thermodynamics.

Abstract: In this contribution we describe the recent progress in the geometric description of equilibrium thermodynamics both in the static and the dynamic setting. A special emphasis is placed on the interplay between the mathematical and physical interpretation of the obtained results. In particular, it is shown that some mathematically sound constructions may fail to agree with the physical evidence. Two particular topics are considered: the metric structure of equilibrium energy manifold and the dynamics of equilibrium thermodynamic processes. A novel approach to the description of temporal evolution of thermodynamic systems is presented. A couple of examples, starting from the stability analysis of a composite thermodynamic system through the modeling of thermodynamic engines to the description of first order phase transitions are intended to demonstrate the use of this approach. In conclusion, we will discuss some open problems and outline the directions for the future research.

Jan Gutt (CFT PAN, Warsaw)
Title: Some conjectures on the geometry of a long snake.

Abstract: I would like to state a couple of conjectures related to a particular non-holonomic mechanical system. They are motivated and supported by compelling evidence, although attempts at a general proof have thus far been unsuccessful. Nevertheless, assuming the conjectures one is led to an interesting and – to my knowledge – new geometric picture, formally describing the kinematics in terms of a (twisted) affine Grassmannian. The talk will introduce the system under consideration, explain the conjectures and explore their consequences. I will also share some details of the current approach to the proof, in hope of soliciting new ideas.

Philippe Jouan (Université de Rouen)

Title: Almost-Riemannian geometry on Lie Groups.

Abstract: A vector field on a connected Lie group G is said to be linear if its flow is a one parameter group of automorphisms (also called infinitesimal automorphism), and a controlled system on G is said to be linear if the drift vector field is linear, and the controlled ones are right invariant. The motivation for dealing with such systems is twofold. On the one hand they are natural extensions of invariant systems on Lie groups. On the other one they can be generalized to homogeneous spaces and appear as models for a wide class of systems, on account of the Equivalence Theorem of [1].

It is natural to define an Almost-Riemannian Structure, ARS in short, on a connected Lie group G by $n = dimG$ invariant or linear vector fields, considered as an orthonormal frame. This kind of ARS can also be defined on homogeneous spaces, and we show that a general ARS that generates a finite dimensional Lie algebra is diffeomorphic to an ARS on a Lie group or a homogeneous space. This equivalence is local or global according to some technical assumptions. Excepted for the equivalence statement, we will describe ARSs on Lie groups defined by one linear vector field and $n - 1$ invariant ones. This description includes the singular locus, the lifting to a sub-Riemannian problem on a $(n + 1)$-dimensional Lie group, the Hamiltonian equations of the geodesics, and is applied to examples.

[1] Ph. Jouan Equivalence of Control Systems with Linear Systems on Lie Groups and Homogeneous Spaces, ESAIM: Control Optimization and Calculus of Variations, 16 (2010) 956-973.

Michał Jóźwikowski (IM PAN, Warsaw)

Title: A contact covariant approach to the Pontryagin Maximum Principle.

Abstract: In the talk I will present an interpretation of the Pontryagin Maximum Principle in the language of contact (instead of symplectic) geometry. I will show its applications to the study of abnormal geodesics in subriemannian geometry. The talk is based on a joint work with prof. Witold Respondek.

Matthias Kawski (Arizona State University)

Title: Combinatorial Hopf Algebras in Nonlinear Control.

Abstract: Efforts to abstract fundamental algebraic structures that underlie the geometry of controlled dynamical systems have a long history. Best known is the classic step from linear to smooth nonlinear that went from matrix linear algebra to Lie algebras. The next step utilizes combinatorial Hopf algebras. We survey how these come into play, how they (re)connect nonlinear control with various other areas of mathematics, and then focus on recent progress. This include new insights into underlying geometric structures, and more efficient computational schemes.

Krzysztof Kozłowski (Poznań University of Technology)

Title: Application of transverse functions in control of nonholonomic systems in robotics.

Abstract: The aim of this paper is to outline applications of the transverse function approach to select mechanical structures met in robotics. we consider practical aspects of these techniques and discuss their advantages and drawbacks. We take into account three particular nonholonomic systems: the differentially driven skid-steering robot with perturbed unicycle kinematics, the two-link planar robot with rotational joint (considered in the flight space), the three-link planar nonholonomic manipulator equipped with the ball gears.

Wojciech Kryński (IM PAN, Warsaw)
Title: Invariants of sub-Riemannian and sub-Lorentzian structures on contact manifolds.

Abstract: We consider general sub-pseudo-Riemannian structures on contact manifolds of arbitrary dimensions. We construct curvature invariants of the structures and characterise the flat structures on the Heisenberg groups. We also show that under additional conditions the structures give rise to Einstein-Weyl geometry in dimension 3.

Andrew Lewis (Queen's University)
Title: Tautological control systems.

Abstract: This talk addresses the problem of feedback invariance in geometric control theory by overviewing a framework that is inherently feedback-invariant. Crucial to the coherence of the method are suitable topologies on spaces of vector fields. The formulation also makes reference to the theory of presheaves and sheaves. While the work is addressed squarely at a fundamental problem of control theory, the technical background for the methods expounded upon are sometimes very complex. For this reason, the intent of the talk is to be expository, rather than technical.

Lev Lokutsievskiy (Lomonosov Univ. Moscow)
Title: On new phenomenon of chaotic behaviour of extremals in problems affine on control.

Piotr Mormul (University of Warsaw, Warsaw, Poland)
Title: Local SR minimizers for Goursat distributions have no corners.

Abstract: In the end of 2013 a draft text [2] appeared, which was settling **in the negative** the Agrachev&Gauthier' SR example of 2012. By merging author's old techniques with those of [2], as well as with the [by-now-classical] technique of Agrachev [1], it is possible to get rid of **all** abnormal extremals' corners in the SR minimizers [for the SR Goursat structures] in any fixed stage of the Goursat Monster Tower (GMT). It is an improvement over a partial negative result reported during the recent IHP trimester. Then the corner points of abnormals that were to be checked for minimality had to be tangential in the sense of GMT. Now they are as general as only possible on abnormal curves of the Goursat structures living in the GMT – just critical in the sense of the GMT. Criticality subsumes tangentiality, yet the difference is enormous. This new **negative** result in the SR geometry is joint with A. A. Agrachev, whose technique in [1] proves very handy in a reduction of dimensions procedure resembling the one of the authors of [2]. Moreover, the nilpotent approximations of SR Goursat structures that are at work, give as a byproduct the absence of corners in SR minimizers living in a new class of distributions – the nilpotent approximations of Goursat germs. Around the non-tangential points they **all** – conjecturally – **are not** Goursat.

References:
[1] A. A. Agrachev; On the equivalence of different types of local minima in sub-riemannian problems, in: Proceedings of the 37th CDC Conference (1998).
[2] E. Le Donne et al; A short proof of the non-minimality of corners in a 4- dimensional subriemannian structure, November 2013 (a draft).

A. Stephen Morse (Yale University)
Title: Problems in Distributed Consensus and Formation Control.

Abstract: Over the past decade there has been growing in interest in distributed control problems of all types. Among these are consensus and flocking problems, and distributed formation control problems. The aim of this two-part lecture is to outline what some of these problems are and to discuss their solutions. Related concepts from spectral graph theory, rigid graph theory, and nonhomogeneous Markov chain theory will be covered.

In the first lecture (joint work with S. Mou and J. Liu) we will present graph-theoretic results appropriate for the analysis of a variety of consensus problems cast in dynamically changing environments. We will discuss several semi-norms which are useful in studying the convergence of infinite sequences of products of stochastic matrices. The concepts of rooted, strongly rooted, and neighbor-shared graphs will be defined, and conditions will be given for compositions of sequences of directed graphs to be of these types. As an illustration of the use of the concepts covered, we will describe a distributed algorithm for solving a linear algebraic equation of the form $Ax = b$ where A is any matrix for which the equation has at least one solution. The equation is simultaneously solved by m agents assuming each agent knows only a subset of the rows of the partitioned matrix $[A\ b]$, the current estimates of the equation's solution generated by its neighbors, and nothing more. Each agent recursively updates its estimate of a solution by utilizing the current estimates generated by each of its neighbors. Neighbor relations are characterized by a time-dependent directed graph $N(t)$ whose vertices correspond to agents and whose arcs depict neighbor relations. By using a suitably defined "mixed matrix norm" we will be able to state necessary and sufficient conditions on the sequence of graphs $N(t)$, $t = 1, 2, \ldots,$ for the algorithm to cause all agents' estimates to converge exponentially fast to the same solution to $Ax = b$.

In the second lecture (joint work with S. Mou, M. A. Belabbas, Z. Sun, and B. D. O. Anderson) we will overview several recent results concerned with the maintenance of formations of mobile autonomous agents eg robots based on the idea of a rigid framework. We will talk briefly about certain classes of "directed" formations for which there is a moderately complete methodology. We then turn to "undirected" formations which is the main focus of this presentation. By an undirected rigid formation of mobile autonomous agents is meant a formation based on "graph rigidity" in which each pair of "neighboring" agents i and j is responsible for maintaining the prescribed distance d_{ij} between them. Recent research by others has led to the development of an elegant potential function based theory of formation control which provides gradient laws for asymptotically stabilizing a large class of rigid, undirected formations in two-dimensional space assuming all agents are described by kinematic point models. The main purpose of this talk is to explain what happens if neighboring agents i and j using such gradient controls have slightly different understandings of what the desired distance d_{ij} between them is suppose to be. The question is relevant because no two positioning controls can be expected to move agents to precisely specified positions because of inevitable imprecision in the physical comparators used to compute the positioning errors. The question is also relevant because it is mathematically equivalent to determining what happens if neighboring agents have differing estimates of what the actual distance between them is. In either case, what one would hope for would be a gradual distortion of the formation from its target shape as discrepancies in desired or sensed distances increase. While this is observed for the gradient laws in question, something else quite unexpected happens at the same time. In this talk we will describe what occurs and explain why. The robustness issues raised here have broader implications extending well beyond formation maintenance to the entire field of distributed estimation and control. In particular, this research illustrates that when assessing the efficacy of a particular distributed algorithm, one must consider the consequences of distinct agents having slightly different understandings of what the values of shared data between them is suppose to be. For without the protection of exponential stability/convergence, it is likely that such discrepancies will cause significant misbehavior to occur.

Florentina Nicolau (INSA Rouen)

Title: Multi-input Control-affine Systems Linearizable via One-fold Prolongation and their Flatness.

Abstract: We study flatness of multi-input control-affine systems. We give a geometric characterization of systems that become static feedback linearizable after a one-fold prolongation of a suitably chosen control. They form a particular class of flat systems. Namely those of differential weight $n + m + 1$, where n is the dimension of the state-space and m is the number of controls. We propose conditions (verifiable by differentiation and algebraic operations) describing that class, construct normal forms and provide a system of PDE's giving all minimal flat outputs. We illustrate our results by several examples.

Paweł Nurowski (CFT PAN, Warsaw)

Title: Rolling without slipping or twisting: from old to new.

Jean-Baptiste Pomet (INRIA Sophia Antipolis)

Title: On averaging techniques in control, Finsler geometry and low thrust orbital transfer

Abstract: This talk will recall some facts on averaging in control with or without the optimal control goal in mind. We then prove a special property of the average systems obtained this way for low thrust orbital transfer in space mechanics.

Jérémy Rouot (INRIA Sophia Antipolis)

Title: Averaging techniques in the time minimal transfer using low propulsion.

Abstract: In this talk we present new results about non coplanar orbital transfer using low propulsion and averaging methods. We combine geometric optimal control and numerical simulations using the Hampath Code.

Yu.L. Sachkov (Program Systems Institute RAS, Pereslav Zaleski)

Title: Sub-Riemannian minimizers, spheres, and cut loci.

Abstract: We will describe a method of evaluation of cut time in sub-Riemannian problems via analysis of symmetries of the exponential mapping and the corresponding Maxwell points. By virtue of this method, we have recently described optimal synthesis, sub-Riemannian minimizers, spheres, and cut loci for left-invariant problems on the Lie group $SH(2)$ of hyperbolic motions of the plane, and on the Engel group.

Andrei Sarychev (University of Florence)

Title: Ensemble Controllability by Lie algebraic methods.

Abstract: Over the last decade there have been a rise of interest and of research regarding controllability of ensembles (parameterized families) of nonlinear control systems

$$\dot{x} = f^{\theta}(x, u), \theta \in \Theta \subset \mathbb{R}^{\nu}$$

by a single θ-independent control $u(\Delta)$. Such problems emerge for example, from a necessity to control a system with "structured uncertainty", when some parameters of the system are subject to "dispersion". In the current presentation (based on joint work with A. Agrachev and Yu. Baryshnikov) we seek to reintroduce in a systematic way the Lie algebraic approach to ensembles of nonlinear systems. We concentrate on approximate ensemble controllability by means of controls of fixed finite dimension. On the contrast to the previous work we advocate a mixed approach, which combines use of Lie brackets, related to Taylor series in state variables, with Fourier-type series in θ. It turns out that a convenient notion for the formulation of the approximate ensemble controllability criteria is the one of the frame in Hilbert space. We start with finite ensembles to which the Lie rank criteria of exact controllability can be applied after proper modification. Two examples of continual ensembles are studied. First is a model example of an ensemble in $\mathbb{R}^3$. Second object of study is a general ensemble of r-dimensional distributions (control-linear systems) on a manifold. In both cases we manage to formulate sufficient and necessary approximately controllability criteria in terms of Lie algebraic frame.

Adam Sawicki (MIT, Cambridge and CFT PAN, Warsaw)
Title: Universality of beamsplitters and control theory.

Abstract: I will show how one can prove universality of a real beamsplitter in quantum optics using some fundamental theorems from control theory and a few magic formulas valid for $SO(N)$ group. This is unpublished work in progress.

Mario Sigalotti (INRIA Saclay)
Title: Sub-Finsler geometry from the viewpoint of optimal control: low-dimensional examples.

Abstract: We study sub-Finsler geometry from the point of view of time-optimal control. In particular, we consider non-smooth and non-strictly convex sub-Finsler structures associated with the Heisenberg, Grushin, and Martinet distributions. Motivated by problems in geometric group theory, we characterize extremal curves and we discuss their optimality and the regularity of metric spheres.

Hector Sussmann (Rutgers University)
Title: A regularity theorem for minimizers of real-analytic subriemannian metrics

Abstract: We prove, for real-analytic subriemannian metrics, that if a trajectory parametrized by arc-length is an arc-length minimizer, then the trajectory is real-analytic on an open dense subset of its interval of definition. The proof is by induction on the fiber dimension, and is based on constructing a subanalytic stratification of the cotangent bundle of the state space compatible with the "abnormal set", i.e., the set of all nonzero covectors that belong to the annihilator of the space of admissible velocity vectors.

Alexander Zuyev (Max Planck Institute for Dynamics of Complex Technical Systems)
Title: Exponential stabilization of nonholonomic systems by using time-varying feedback controls.

Abstract: This lecture is devoted to the stabilization problem for nonlinear driftless control systems by means of a time-varying feedback control. It is assumed that the vector fields of the system together with their first order Lie brackets span the whole tangent space at the equilibrium. A family of trigonometric open-loop controls is constructed to approximate the gradient flow associated with a Lyapunov function. These controls are applied for the derivation of a time-varying feedback law under the sampling strategy. By using Lyapunov's direct method, we prove that the controller proposed ensures exponential stability of the equilibrium. We present some examples where such control design procedure is applied for the stabilization of nonholonomic mechanical systems.