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Martingale inequalities and Fourier multipliers

For a given bounded measurable function $m : \mathbf{R}^d \to \mathbf{C}$, let T_m denote the associated Fourier multiplier on $\mathbf{R}^d$, i.e., the linear operator given by the identity $\widehat{T_m f} = m\widehat{f}$ between Fourier transforms. By Plancherel's theorem, T_m defines a bounded operator on $L^2(\mathbf{R}^d)$. A classical problem of harmonic analysis is to study conditions on a symbol m which imply that the multiplier T_m extends to a bounded operator on $L^p(\mathbf{R}^d)$; there has also been considerable interest in the study of the action of T_m in other spaces (e.g., $L^p(\mathbf{R}^d) \to L^{p,\infty}(\mathbf{R}^d)$, $L \log L(\mathbf{R}^d) \to L^1(\mathbf{R}^d)$, etc.).

The purpose of the talk is to present a probabilistic approach to the above problem, which rests on the careful application of inequalities for differentially subordinated martingales. The argument enables obtaining precise and quite general estimates for the special class of the so-called Lévy multipliers. This class, introduced by Bañuelos, Bielaszewski and Bogdan in [1], comes from an appropriate modulation of jumps of certain Lévy processes and contains several important examples. In particular, as will be illustrated during the talk, the above approach yields interesting sharp estimates for second-order Riesz transforms and the Beurling-Ahlfors operator.

The talk will be mostly based on the results from the papers [2, 3, 4, 5, 6].

References

[1] Rodrigo Bañuelos, Adam Bielaszewski and Krzysztof Bogdan, *Fourier multipliers for non-symmetric Lévy processes.* Marcinkiewicz centenary volume, 9–25, Banach Center Publ., 95, Polish Acad. Sci. Inst. Math., Warsaw, 2011.

[2] Rodrigo Bañuelos and Adam Osękowski, *On Astala's theorem for martingales and Fourier multipliers*, submitted.

[3] Adam Osękowski, *Logarithmic inequalities for Fourier multipliers*, Math. Z. 274 (2013), no. 1-2, 515–530.

[4] Adam Osękowski, *Sharp localized inequalities for Fourier multipliers*, Canad. J. Math. 66 (2014), no. 6, 1358–1381.

[5] Adam Osękowski, *On restricted weak-type constants of Fourier multipliers*, Publ. Mat. 58 (2014), no. 2, 415–443.

[6] Adam Osękowski, *Weak-type inequalities for Fourier multipliers with applications to the Beurling-Ahlfors transform*, J. Math. Soc. Japan 66 (2014), no. 3, 745–764.