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Recent developments in random affine recursions

Joint work with Dariusz Buraczewski, Jacek Zienkiewicz, Rafał Latała, Piotr Nayar and Tomasz Tkocz.

We consider the following affine recursion in $\mathbb{R}^d$

$$X_n = A_n X_{n-1} + B_n, \quad n \geq 1 \tag{1}$$

where (A_n, B_n) is a sequence of i.i.d. (independent identically distributed) random variables with values in $GL(\mathbb{R}^d) \times \mathbb{R}^d$ and $X_0 \in \mathbb{R}^d$ is the initial distribution. The generic element of the sequence (A_n, B_n) will be denoted by (A, B). Under mild contractivity hypotheses the sequence X_n converges in law to a random variable R, which is the unique solution of the stochastic difference equation

$$R =_d AR + B, \qquad \text{where } R \text{ is independent of } (A, B) \tag{2}$$

and equality is meant in law. The main issues concerning (1) are characterization of the tail of R, regularity of the law of R, behavior of iterations X_n.

First results were obtained already in seventies by Kesten for matrices with positive entries and by Grincevicius and Vervaat in the one dimensional case. However recently, due to its importance, equation (2) has again attracted attention of many people the contribution of the Wrocław team being essential.

With so called Kesten assumptions R has a heavy tail behavior, which means that there is $\alpha > 0$ such that $\lim_{t\to\infty} \mathbb{P}\{\|R\| > t\} t^\alpha = C_\infty > 0$. However, there is still no satisfactory description of the constant C_∞ as well as the rate of convergence of $\mathbb{P}\{\|R\| > t\} t^\alpha$ to C_∞. *I am going to talk about the latter problem both in the case of recursion* (1) *and the Lipschitz iterative systems modeled on it.* Good formulae for C_∞ are important from the point of view of applications.

The talk is based on the joint work with Dariusz Buraczewski and Jacek Zienkiewicz (University of Wrocław), Rafał Latała and Piotr Nayar (University of Warsaw), and Tomasz Tkocz (University of Warwick).