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Non-local Hamilton–Jacobi–Bellman equations on Hilbert spaces

The presentation is devoted to the existence and uniqueness problems for the following HJB equation on a Hilbert space H:

$$\frac{\partial u}{\partial t}(t,x) + \inf_{a\in\Lambda}(\mathcal{L}^a u(t,x) + f(x,a)) = 0, \;\; u(T,x) = g(x),\, x \in H,\, t \in [0,T]. \quad (1)$$

For a fixed a in a set of parameters Λ, $\mathcal{L}^a$ is an integro-differential operator and $f(\cdot,a)$ is a real function on H. The operator $\mathcal{L}^a$ is generated by solutions of the following stochastic evolution equation

$$dX(s) = (AX(s) + b((X(s),a))ds + B(X(s-),a)dL(s), \quad (2)$$

where A is a dissipative operator on H, in general unbounded, $b(\cdot,a)$, $B(\cdot,a)$ are transformations from H into H and from $[0,T]$ into the space of linear operators acting between Hilbert spaces U and H. Moreover L is a Lévy process on U with the Lévy measure ν. Formally $\mathcal{L}^a$ acts on a function v as follows

$$\mathcal{L}^a v(x) =< Ax, Dv(x) > + < b(x,a), Dv(x) >$$

$$+ \int_U (v(x + B(x,a)u) - v(x) - < Dv(x), B(x,a)u >)\nu(du).$$

We will present conditions under which the equation (1) has unique viscosity solution. We also identify the solution with the so called value function

$$u(t,x) = \inf_{a(\cdot)}\mathbb{E}(\int_t^T f(X(s),a(s))ds + g(X(T))) \quad (3)$$

The inf operation in (3) is with respect to control processes $a(\cdot)$, taking values in Λ and the corresponding process X starts from x at moment t and is solving (2) with a replaced by $a(s)$. Our theorems cover HJB equations for controlled stochastic wave equations.

The results were obtained in collaboration with A. Święch.

References

[1] A. Święch and J. Zabczyk, *Uniqueness for integro-PDE in Hilbert spaces.* Potential Anal. 38(2013), no.1, 233-250.

[2] A. Święch and J. Zabczyk, *Integro-PDE in Hilbert spaces: Existence of viscosity solutions*, in preparation.