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Analytical, geometric, and stochastic properties of a class of infinite dimensional stochastic processes with unbounded diffusion

Joint work with John Karlsson.

We give a review of properties of Dirichlet forms and stochastic processes related to positive symmetric bilinear forms defined on certain cylindrical functions by $\mathcal{E}(F,G) = \int \langle DF, ADG\rangle_{\mathbb{H}}\, \varphi\, d\nu$. Here ν is the Wiener measure on the path space $P_{m_0} := \{\gamma \in C([0,1); M) : \gamma(0) = m_0\}$ over an in general non-compact Riemannian manifold M, the function φ is a non-negative weight, and A is an in general ν-a.e. unbounded operator in the Cameron-Martin space $\mathbb{H}$. Among other things, we are interested in the rate of increase of λ_i, $i \in \mathbb{N}$, if we suppose

$$\mathcal{E}(F,G) = \int \Big\langle DF, \sum_{i=1}^{\infty} \lambda_i \, \langle S_i, DG\rangle_{\mathbb{H}}\, S_i \Big\rangle_{\mathbb{H}} \varphi\, d\nu$$

where S_i, $i \in \mathbb{N}$, is the ONB in $\mathbb{H}$ consisting of the Schauder functions. Particular attention is given to weight functions φ of the form

$$\varphi(\gamma) = \exp\left\{ \int_0^1 \langle V(\gamma_t),\, d\gamma_t\rangle_{T(\gamma(t))} - \frac{1}{2}\int_0^1 |V(\gamma_t)|^2_{T(\gamma(t))}\, dt \right\}$$

as φ is then the Radon-Nikodym derivative of some measure with respect to ν, corresponding to the distribution of the diffusion process on M with generator $\frac{1}{2}\Delta_M + V$. Elements of the related stochastic calculus are presented.

The talk is based on the following papers.

References

[1] John Karlsson and Jörg-Uwe Löbus. A class of infinite dimensional stochastic processes with unbounded diffusion. To appear in Stochastics, http://www.tandfonline.com/doi/full/10.1080/17442508.2014.959952

[2] Jörg-Uwe Löbus. A class of processes on the path space over a compact Riemannian manifold with unbounded diffusion. Trans. Amer. Math. Soc. 356(9), 3751-3767, 2004.

[3] Feng-Yu Wang and Bo Wu. Quasi-regular Dirichlet forms on Riemannian path and loop spaces. Forum Math. 20(6), 1085-1096, 2008.