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Annealed and quenched asymptotics for semigroups of Lévy processes with Poissonian interaction

Joint work with Kamil Kaleta.

Consider a Lévy process X in $\mathbb{R}^d$, with generator L, whose evolution is affected by a Poisson-type random potential V^ω.

We are interested in asymptotical behaviour, as $t \to \infty$, of solutions of the so-called spatially continuous parabolic Anderson problem driven by L :

$$\partial_t u = Lu - V^\omega u, \quad u^\omega(0, x) \equiv 1.$$

We consider processes whose characteristic exponent in its Lévy-Khinchine representation is, for small arguments, close to the characteristic exponent of a symmetric $\alpha-$stable process, $\alpha \in (0, 2]$. The Poissonian potential V^ω is assumed to have a compact profile. For such processes it is known (Donsker-Varadhan, Okura) that the averaged ('annealed') behaviour of solutions depends just on the parameter α:

$$\lim_{t\to\infty} \frac{\log \mathbb{E}_{\mathbb{Q}} [u^\omega(t, x)]}{t^{d/(d+\alpha)}} = -K = K(X). \tag{1}$$

We investigate the pointwise ('quenched') behaviour of the solutions $u^\omega(t, x)$. This behaviour depends deeply on the Lévy measure of the process. For every such a process, we prove that there exist a rate function $g(t)$, two 'correction' functions $c_1(t)$, $c_2(t)$, and two constants $K_1, K_2 > 0$ such that:

Theorem 1. *For all $x \in \mathbb{R}^d$, $\mathbb{Q}-$almost surely*

$$\limsup_{t\to\infty} \frac{\log u^\omega(t, x) + c_1(t)}{g(t)} \le -K_1,$$

and

$$\liminf_{t\to\infty} \frac{\log u^\omega(t, x) + c_2(t)}{g(t)} \ge -K_2.$$

For particular processes the correction terms might not be needed. Our approach covers e.g. the following examples:

• symmetric $\alpha-$stable processes: $g(t) = t^{d/(d+\alpha)}$, $c_1(t), c_2(t) = 0$, (same annealed and quenched rate)

• the relativistic process: $g(t) = t/(\log t)^{d/2}$, $c_1(t) = 0$, $c_2(t) = 0$ (the quenched asymptotics different from the annealed), additionally, we establish that $K_1 = K_2$ i.e. the limit exists. Both the quenched and the annealed rate coincide with that for the Brownian motion.

Other rates of quenched asymptotics are possible as well.

References

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