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## Local fluctuations of critical Mandelbrot cascades

*Joint work with Dariusz Buraczewski and Piotr Dyszewski.*

A lognormal multiplicative chaos was introduced by Mandelbrot to model turbulences. Its "toy model", so-called Mandelbrot cascades, is a measures valued stochastic process. An appropriately normalized sequence of multiplicative cascades converges weakly in probability to a nontrivial limit measure $\mu$. In the simplest example, given $\beta > 0$ (*inverse temperature parameter*), $\mu$ is a finite random measure on the unit interval $I = [0, 1)$ satisfying the self-similarity property

$$\mu(B) \stackrel{d}{=} \frac{e^{\beta\xi_1 - \beta^2/2}}{2}\mu_1\Big(2\big(B \cap [0, 1/2)\big)\Big) + \frac{e^{\beta\xi_2 - \beta^2/2}}{2}\mu_2\Big(2\big(B \cap [1/2, 1)\big) - 1\Big),$$

where $\mu_1, \mu_2$ have the same law as $\mu$, $\xi_1, \xi_2$ are standard normal variables and all random variables $\mu_1, \mu_2, \xi_1, \xi_2$ are independent.

It is known that there is a critical value $\beta_c = \sqrt{2 \log 2}$ (*freezing temperature*) where there is a phase transition. For $\beta \le \beta_c$ (*high temperature*) the measure $\mu$ is continuous, although singular with respect to the Lebesgue measure, whereas for $\beta > \beta_c$ (*low temperature*) it is purely atomic. In the continuous case one may asked how the measure $\mu$ fluctuates. We are looking for deterministic functions $\phi$ and $\psi$ such that for almost all realizations of $\mu$, for $\mu$-almost all $x \in I$ and for sufficiently large $n$ we have

$$\phi(n) \le \mu(B_n(x)) \le \psi(n),$$

where $B_n(x)$ is a dyadic set of length $2^{-n}$ containing $x$.

The question above has been answered in the subcritical case $\beta < \beta_c$ by Liu [1]. For the critical case $\beta = \beta_c$ some partial results were obtained by Barral, Kupiainen, Nikula, Saksman and Webb [2]. In the talk we will present sharper results.

### References

[1] Q. Liu *On generalized multiplicative cascades.* Stoch. Proc. Appl. 86, 263–286 (2000).

[2] J. Barral, A. Kupiainen, M. Nikula, E. Saksman & C. Webb *Critical Mandelbrot Cascades.* Commun. Math. Phys. 325, 68–711 (2014).