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Modified Hardy–Littlewood maximal operators on nondoubling metric measure spaces

We investigate the modified Hardy-Littlewood maximal operators, uncentered $M_k f(x) = \sup_{x\in B} \frac{1}{\mu(kB)} \int_B |f| d\mu$, and centered $M_k^c f(x) = \sup \frac{1}{\mu(B(x,kr))} \int_{B(x,r)} |f| d\mu$, $k \geq 1$, in the setting of a general metric measure space (X, d, μ). The following result enhances the 'basic covering theorem', [1, Theorem 1.2].

Theorem 1. *Let (X, d) be an arbitrary metric space and let $\tau > 3$ be fixed. Every family $\mathcal{F}$ of balls of uniformly bounded radii contains a disjointed family $\mathcal{G}$ such that*

$$\bigcup_{B\in\mathcal{F}} B \subset \bigcup_{B\in\mathcal{G}} \tau B.$$

This result is sharp in the sense that $\tau = 3$ is not enough. Theorem 1 is the main ingredient in the proof of the main result, Theorem 2, which may be seen as an enhancement of the results of Nazarov, Treil and Volberg [3] (the case of a separable metric space and the centered maximal operator), Tolsa [6] (the case of $X = \mathbb{R}^n$, $d = d_\infty$), Sawano [4] (the case of a separable locally compact metric space and a Radon measure), and Hytönen [2] (the case of a geometrically doubling metric space).

Theorem 2. *The maximal operators M_3 and M_2^c are of weak type $(1, 1)$ with the weak type constants equal to one.*

This result is sharp in the sense that, in general, any $k < 3$ or any $k < 2$ is not enough in the uncentered or in the centered case, respectively, for M_k or M_k^c to be of weak type $(1, 1)$.

References

[1] J. Heinonen, *Lectures on analysis on metric spaces*, Universitext, Springer, 2001.

[2] T. Hytönen, *A framework for non-homogeneous analysis on metric spaces and the RBMO space of Tolsa*, Publ. Mat. 54 (2010), 484-504.

[3] F. Nazarov, S. Treil, and A. Volberg, *Weak type estimates and Cotlar inequalities for Calderón-Zygmund operators on nonhomogeneous spaces*, Internat. Math. Res. Notices 9, (1998), 463-487.

[4] Y. Sawano, *Sharp estimates of the modified Hardy Littlewood maximal operator on the nonhomogeneous space via covering lemmas*, Hokkaido Math. J. 34 (2005), 435-458.

[5] K. Stempak, *Modified Hardy-Littlewood maximal operators on nondoubling metric measure spaces*. Ann. Acad. Sci. Fenn. Math. 40 (2015), 443-448.

[6] X. Tolsa, BMO, *H^1, and Calderón-Zygmund operators for non doubling measures*, Math. Ann. 319 (2001), 89-149.