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HITTING TIME DISTRIBUTIONS OF SYMMETRIC LÉVY PROCESSES

Joint work with Tomasz Grzywny.

Let X_t be a symmetric Lévy process on \mathbb{R}^d . We investigate the distribution of the first hitting time of the ball of radius R centered at 0, $T_{B_R} = \inf\{t > 0; |X_t| \leq R\}$. For isotropic unimodal processes, under some weak type scaling properties, which are expressed in terms of the characteristic exponent ψ , we provide sharp estimates of the density function of $P^x(T_{B_R} \leq t)$. The uniform estimates are valid for all starting points $|x| > R, t > 0$ and R . The method of estimation relies on recent estimates of the heat kernel of the killed process upon hitting a ball of radius R [1]. We also give some estimates of the density of the hitting time of a compact sets with smooth boundary.

In the one-dimensional case we also consider the problem of hitting times of points or intervals. Let T_0, T_I be the hitting times of 0 or an interval I , respectively. Under the assumption that the process hits points with probability one and certain assumptions on the compensated potential kernel or the characteristic exponent we provide the optimal estimates of the tail function $P^x(T_0 \geq t)$ or $P^x(T_I \geq t)$ and its limiting behaviour as $t \rightarrow 0$. If we additionally assume unimodality of the process we also obtain sharp estimates of the density function. Similar results, for the hitting time of 0, were obtained by Juszczyzsyn and Kwaśnicki [2] but with much more restricted assumptions than ours and by different methods. One of the main results regarding one-dimensional symmetric Lévy processes reads as follows.

Theorem 1. *Let $1 < \alpha \leq 2$. If $\frac{\psi(\xi)}{\xi^\alpha} \approx h(\xi)$, $\xi > 0$, where $h(\xi)$ is non-decreasing then for $x \in \mathbb{R}$ and $t > 0$,*

$$P^x(T_0 > t) \approx \frac{K(x)}{K(1/\psi^{-1}(1/t))} \wedge 1 \approx \frac{1}{t\psi^{-1}(1/t)|x|\psi(1/x)} \wedge 1,$$

where $K(x)$ is the compensated potential kernel and ψ^{-1} is the generalized inverse of the characteristic exponent ψ .

REFERENCES

- [1] K. Bogdan, T. Grzywny and M. Ryznar *Dirichlet heat kernel for unimodal Lévy processes*. Stochastic Process. Appl. vol.124 no. 11 p. 3612–3650 , 2014.
- [2] T. Juszczyzsyn, M. Kwaśnicki, *Hitting times of points for symmetric Lévy processes with completely monotone jumps* - preprint arXiv:1403.3714, 2014