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Homogenization of periodic diffusions with small jumps

In this talk, we will discuss the problem of homogenization of a class of diffusions with jumps, that is, Feller processes generated by integro-differential operators of the following type

$$\begin{aligned}\mathcal{A}f(x) =&\langle b(x), \nabla f(x)\rangle + \frac{1}{2}\operatorname{div} c(x)\nabla f(x)\\ &+ \int_{\mathbb{R}^d} \left(f(y+x) - f(x) - \langle y, \nabla f(x)\rangle 1_{\{z:|z|\leq 1\}}(y)\right)\nu(x, dy).\end{aligned}$$

Under the assumptions that the underlying diffusion with jumps (i) has periodic coefficients, (ii) the corresponding drift function vanishes, (iii) it admits only "small jumps" (that is, $\sup_{x\in\mathbb{R}^d}\int_{\mathbb{R}^d}|y|^2\nu(x,dy) < \infty$) (iv) and under certain additional regularity conditions, we prove that the homogenized process is a Brownian motion. This problem is closely related to the problem of homogenization of boundary value problems arising in studying the behavior of heterogeneous media. The presented results generalize the classical and well-known results related to the homogenization of diffusions.

References

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