Paweł Wolff
University of Warsaw and Polish Academy of Sciences, Poland

Gaussian kernels have also Gaussian minimizers

Joint work with Franck Barthe.

The title alludes to the paper of Lieb [2] on maximizers of a multilinear functional with a Gaussian kernel acting on a product of L^p spaces. His result identifies functions $f_i \in L^{p_i}(\mathbb{R}^{n_i})$ which maximize

$$J(f_1, \ldots, f_m) = \frac{\int_{\mathbb{R}^n} e^{-\langle Qx,x\rangle} \prod_{i=1}^m f_i(B_i x)\, dx}{\prod_{i=1}^m \left(\int_{\mathbb{R}^{n_i}} |f_i|^{p_i} \right)^{1/p_i}},$$

where $Q\colon \mathbb{R}^n \to \mathbb{R}^n$ is positive semi-definite and $B_i\colon \mathbb{R}^n \to \mathbb{R}^{n_i}$ are surjective linear maps.

In this talk we shall address a problem of identifying minimizers of J among *positive* functions f_i in case some of the indices p_i are below 1. Our result is related to several "reversed" inequalities known in the literature including the reversed Young convolution inequality, Borell's reversed hypercontractivity and the reversed Brascamp-Lieb inequalities.

Based on a joint work [1] with Franck Barthe from the University of Paul Sabatier, Toulouse.

References

[1] F. Barthe, P. Wolff. Positivity improvement and Gaussian kernels. *C. R. Acad. Sci. Paris Sér. I Math.*, 352 (12):1017–1021, 2014.

[2] E. H. Lieb. Gaussian kernels have only Gaussian maximizers. *Invent. Math.*, 102 (1):179–208, 1990.