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Fractal Burgers equation with the critical exponent

Let $d \in \mathbb{N}$ and $\alpha \in (1,2)$. We consider the following pseudo-differential equation

$$\begin{cases} u_t - \Delta^{\alpha/2} u + b \cdot \nabla \left(u|u|^{(\alpha-1)/d} \right) = 0, & t > 0,\ x \in \mathbb{R}^d, \\ u(0,x) = M\delta_0(x), \end{cases} \tag{1}$$

where $M > 0$ is an arbitrary constant and $b \in \mathbb{R}^d$ is a constant vector. The existence and some basic properties of the solution $u_M(t,x)$ of this equation were proved in [1]. In [2] the authors proved that for sufficiently small M there is a constant $C = C(d,\alpha,M,b)$ such that

$$u_M(t,x) \le Cp(t,x), \qquad t > 0,\ x \in \mathbb{R}^d. \tag{2}$$

I will present a new method which allows to show pointwise estimates of solutions to the nonlinear problem (1) without the smallness assumption imposed on M. This method has been inspired by the proof of [3, Theorem 1]. The main result is

Theorem 1. *Let $d \ge 1$ and $\alpha \in (1,2)$. Let $u_M(t,x)$ be the solution of the equation (1). There exists a constant $C = C(d,\alpha,M,b)$ such that*

$$C^{-1}p(t,x) \le u_M(t,x) \le Cp(t,x), \qquad t > 0,\ x \in \mathbb{R}^d.$$

The talk is based on the paper [4].

References

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