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Zonal polynomials and a multidimensional quantum Bessel process

Joint work with Marcin Świeca.

The aim of the talk is to present a construction of a classical (commutative) Markov process (the multidimensional quantum Bessel process) which generalizes a construction given by Biane in [1].

Let H be the Heisenberg group realized as the set $H = \mathrm{Sym}(m, \mathbb{C}) \times \mathbb{R}$, where $\mathrm{Sym}(m, \mathbb{C})$ stands for the space of symmetric complex $m \times m$ matrices. We first find a semigroup $(Q_t)_t$ of completely positive contractions on the group C^*-algebra $C^*(H)$ of the Heisenberg group. This semigroup might be viewed as the semigroup of noncommutative Brownian motion on the dual of H. Then we consider the action of the unitary group $U(m)$ on H under which $(U(m) \ltimes H, U(m))$ is a Gelfand pair. This, in particular, means that the convolution subalgebra of the functions on H invariant under the $U(m)$-action, is commutative. After extending the convolution subalgebra to the commutative sub-C^*-algebra $C^*_R(H)$ of $C^*(H)$, we use the spherical functions of $(U(m) \ltimes H, U(m))$ to explicitly find the restriction of $(Q_t)_t$ to $C^*_R(H)$. The restriction, by the Gelfand-Naimark theorem and the Riesz Representation Theorem, is a (classical) Markov semigroup on the spectrum $\sigma(C^*_R(H))$ of $C^*_R(H)$. Next we present a way to embed $\sigma(C^*_R(H))$ into a subset of $\mathbb{R}^{m+1}$ (a multidimensional Heisenberg fan), thus obtaining the classical Markov process.

Since the spherical functions of the Gelfand pair $(U(m) \ltimes H, U(m))$ are expressed in terms of the zonal polynomials (see e.g. [5]), some properties of these polynomials are fundamental for the actual computation of the restriction of $(Q_t)_t$ to $C^*_R(H)$.

One of the properties of the multidimensional quantum Bessel process is that the sum of its coordinates is an example of the so-called quadratic harness (see [2]), namely a bi-1-Poisson process ([3]).

The talk is based on the joint work [6] with Marcin Świeca (Warsaw University of Technology).

References

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