Topics in Analysis and Holomorphic Dynamics (workshop), Warsaw, September 15-19, 2015

Abstracts of talks

Matings of some cubic polynomials

Magnus Aspenberg
(Joint work with Pascale Roesch)

Tan Lei proved that all postcritically finite Newton maps of degree 3 can be described by matings of cubic polynomials or so called captures. She conjectured that in fact all Newton maps (of degree 3) can be described this way. In this talk I will present a proof that this holds for a large class of Newton maps, which are neither are postcritically finite nor hyperbolic.

Free energy jumps up

Neil Dobbs

Free energy is not semi-continuous, but we show that in some contexts, it nearly is. This engenders proofs of existence of equilibrium states and (almost) continuity of equilibrium states as one varies the potential and the map. We do this in the context of one-dimensional dynamics.

Entire curves and surface automorphisms

Christophe Dupont
(Joint work with Serge Cantat)

We deal with holomorphic automorphisms of projective complex surfaces. If such a map has positive entropy then it has a unique measure of maximal entropy, which is the product of two invariant currents. Remarkably, the Pesin stable and unstable manifolds are injective holomorphic images of the complex plane (entire curves). We prove two properties concerning these curves when the maximal measure is absolutely continuous. The first one is a smoothness of the slices of the invariant currents, the second one is a compactness property. These tools allow us to show a rigidity result : the complex surface must be a torus and the automorphism must be a linear Anosov map (up to finite covering and normalization).

Escaping points in the boundary of Baker domains

Núria Fagella
(Joint work with K. Barański, X. Jarque and B. Karpińska)

We study the dynamical behaviour of points in the boundaries of simply connected Baker domains of meromorphic maps of finite degree on U. We show that there is a dichotomy in terms of the harmonic measure of boundary points that escape to infinity under iteration, which is zero or infinity depending of the type of Baker domain. Additionally, we present some extensions to the infinite degree case and explain how the results also apply to basins of attraction of attracting or parabolic fixed points.

Distribution of postcritically finite polynomials: Speed of convergence

Thomas Gauthier

In the moduli space of degree d polynomials, there exists a probability measure which detects the strongest possible bifurcations. I proved it C. Favre that postcritically finite hyperbolic parameters equidistribute this measure when the length of all critical orbit explode. Unfortunately, the proof is relying on a very strong arithmetic theorem.
The aim of this talk is to introduce the bifurcation measure and to present a quantitative version of this result I recently proved with G. Vigny using only complex analytic tools. We will first began with the quadratic family $z^2 + c$ in which the bifurcation measure is the harmonic measure if the Mandelbrot set and then explain the additional difficulties that arise in higher dimensional parameter spaces.

Smooth curves traversing the Mandelbrot set

Jacek Graczyk
(Joint work with Michael Benedicks)

We study the Mandelbrot set along typical smooth curves with respect to the harmonic measure. The work uses both the techniques of Benedicks-Carleson as well as a recent technique of amplification.

Quasicircles of dimension $1 + k^2$ do not exist

Oleg Ivrii

A well-known theorem of Smirnov says that the Hausdorff dimension of a k-quasicircle is bounded above by $1 + k^2$. In this talk, I will show that $D(k) = 1 + \Sigma^2 k^2 + \mathcal{O}(k^{2.5})$, where $D(k)$ is the maximal dimension of a k-quasicircle and Σ^2 is the maximal asymptotic variance of the Beurling transform (taken over the unit ball of L^∞). In view of Hedenmalm's estimate $\Sigma^2 < 1$, this shows that Smirnov's bound is not sharp.

Limit drift
Genadi Levin

We discuss problems of the existence of wild attractors and the measure of Julia sets for some real infinitely-renormalizable maps. Based on a joint work with Greg Świątek *arXiv:1403.4418*.

On quasiconformal (in-) compatibility of satellite copies of the Mandelbrot set
Luna Lomonaco

In the paper 'On the dynamics of polynomial-like mappings' Douady and Hubbard introduced the notion of polynomial-like maps. They used it to identify homeomorphic copies of the Mandelbrot set inside the Mandelbrot set. They conjectured that in case of primitive copies the homeomorphism between the homeomorphic copy of the Mandelbrot set and the Mandelbrot set is q.-c., and similarly in the satellite case, it is q.-c. off any small neighborhood of the root. These conjectures are now Theorems due to Lyubich. The satellite copies of the Mandelbrot set are clearly not q-c homeomorphic to the Mandelbrot set. But are they mutually q-c homeomorphic? In general, this question has a negative answer: For any two satellite copies of the Mandelbrot set, the induced Douady-Hubbard homeomorphism is not q-c, if the root multipliers, which are primitive q and q' roots of unity, have q different from q'.

Real analyticity for random dynamics of transcendental functions
Volker Mayer

This talk concerns real analyticity of dimension for random dynamics of entire or meromorphic functions and is based on ongoing and joint work with M. Urbański and A. Zdunik.

Metric properties of mean wiggly continua
Nicolae Mihalache

We study lower and upper bounds of the Hausdorff dimension for sets which are wiggly at scales of positive density. We obtain optimal dimension estimates using minimal geometric information, based on beta numbers introduced by Peter Jones. Applications in non-uniform dynamical systems will be briefly discussed.

Holomorphic interpolation: from dynamics to analysis
István Prause

I will review instances of holomorphic interpolation. This technique can be used to upgrade natural apriori bounds by exploiting holomorphic dependence through parameter space. The method originates in dynamics but also has found applications in analysis. Topics include variation of dimension of Julia sets, quasiconvexity of energy functionals, rotational phenomena for bilipschitz maps and dimension of quasicircles.

Random nodal portraits
Mikhail Sodin

The theme of talk will be a recent progress in understanding statistics of the number of connected components of the zero sets of random functions of several real variables. The primary examples are various Gaussian ensembles of real-valued polynomials (algebraic or trigonometric) of large degree on the sphere or torus, and translation-invariant smooth Gaussian functions on the Euclidean space restricted to large domains. The talk will be based on a joint work with Fedor Nazarov *arXiv:1507.02017*.

Fine Structure of the Connectedness Locus for Uni-Critical Polynomials
Grzegorz Świątek
(Joint work with J. Graczyk)

The connectedness locus is a generalization of the classical Mandelbrot set in the case of higher degree polynomials with one critical point. I will discuss some contributions to the old problem of describing the similarity between a neighborhood of a point on the boundary of the connectedness locus and the corresponding Julia set. There clearly are limitations on such similarity, so the exact statements have to careful. Loosely speaking, however, for a typical point in the sense of the harmonic measure on the boundary of the connectedness locus we manage to construct a similarity map which is quasiconformal on a ball and conformal, i.e. complex-differentiable with non-zero derivative, at the point itself.

Random dynamics of transcendental meromorphic functions
Mariusz Urbański
(Joint work with V. Mayer)

It concerns random dynamics of transcendental functions $f\colon \mathbb{C} \to \bar{\mathbb{C}}$. We will establish the existence of randmom conformal measures and their invariant versions. An appropriately defined spectral gap property will be shown. In classical situations there is a natural and powerful proof of this property which stems from Birkhoff's contraction principle for operators preserving a positive cone. This method however fails in our non-compact situation. We will nevertheless define appropriate invariant cones of positive functions and will revive an old approach of Bowen to overcome this difficulty.