Structure and classification of C*-algebras

Titles and abstracts

21–25 November 2016

Sara Arklint, University of Copenhagen
The K-theoretical range of all Cuntz-Krieger algebras
22 November, 14:00-15:00

In 2006, Restorff showed that the class of purely infinite Cuntz-Krieger algebras are classified by reduced filtered K-theory, and Eilers-Restorff-Ruiz-Sørensen have now extended the result to all Cuntz-Krieger algebras (even all unital graph C^*-algebras) by augmenting the order of the K_0-groups to Restorff's invariant.

I will describe the range of Cuntz-Krieger algebras (and unital graph C^*-algebras) under these invariants, thus completing the classification, and will note how in the purely infinite case this gives a new characterization of the Cuntz-Krieger algebras.

This is joint work with Bentmann and Katsura.

Caleb Eckhardt, Miami University
A survey of recent classification results related to representations of nilpotent groups
22 November, 15:30-16:30

The class of C*-algebras generated by irreducible representations of finitely generated nilpotent groups (call this class C) were recently shown to be classifiable by their Elliott invariant. We'll discuss this result in this talk. Then we'll focus on the calculation of the Elliott invariant for all of the faithful irreducible representations of the unitriangular subgroup of $GL(\mathbf{Z}, 4)$. It is not clear how the class C sits inside all of the classifiable C*-algebras, i.e. what the range of the Elliott invariant is when restricted to the class C. We'll discuss a few broad restrictions in this direction. Some of this is joint work with Gillaspy, Kleski and/or McKenney.

Søren Eilers, University of Copenhagen
The classification of unital graph C*-algebras
21 November, 14:00-15:30

The ambition to classify graph C*-algebras and their precursors, the Cuntz-Krieger algebras goes back to the inception of the field. In fact, as early as 1981, Enomoto, Fujii and Watatani showed by ad hoc methods how to classify all the simple Cuntz-Krieger algebras that may be described by a 3x3-matrix, predating by more than a decade the classification result of Rørdam covering all simple Cuntz-Krieger algebras.

With the classification problem resolved for all simple graph C*-algebras around the turn of the millenium due to the realization that any such C*-algebra is either AF or purely infinite, the quest for classification of graph C*-algebras moved into the realm of non-simple classification, where it has been a key problem ever since. The endeavor has evolved in parallel with the gradual realization of what invariants may prove to be complete in the case when the number of ideals is finite and the C*-algebras in question are not stably finite. In this sense, the

fundamental results obtained on the classification of certain classes of graph C*-algebras are, one may hope, playing a role parallel to the one played by Rrdam's result as a catalyst for the Kirchberg-Phillips classification.

I will present a complete classification theorem for all unital graph C*-algebras, obtained in joint work with Restorff, Ruiz and Sørensen. Emphasizing the discrete nature of our proof, I will also discuss the difficulties involved in an attack on the general (non-unital) case.

George Elliott, Fields Institute, University of Toronto
On the classification of simple C*-algebras with finite nuclear dimension
21 November, 10:30-12:00

Recently, a complete classification of unital simple (separable) UCT C*-algebras with finite nuclear dimension has been obtained, extending to the finite case the famous axiomatic result of Kirchberg and Phillips in the infinite case. This is a consequence of the work of many authors, over a period of thirty years. Some progress has been made in the non-unital setting. An abstract theorem has been obtained in the special case of trivial K-groups (joint work with Z. Niu). (In this case the tracial invariant is all that is left!) Preliminary results have been obtained in the general case (joint work with B. Jacelon and Niu).

Jamie Gabe, University of Southampton
A new K-theoretic invariant for C^*-algebras
25 November, 15:30-16:30

I will introduce a new K-theoretic invariant for C^*-algebras. The invariant is, in particular, computable for large classes of crossed products by endomorphisms $B \rtimes \mathbb{N}$, e.g. whenever B is an AF-algebra, in which case the invariant is essentially the Pimsner–Voiculescu sequence up to a suitable equivalence relation. The invariant can be used to classify large classes of separable, nuclear, purely infinite C^*-algebras A of real rank zero, with all ideals in the UCT class, in particular when A satisfy one of the following: (i) A has finitely many ideal, (ii) $\operatorname{Prim} A$ is Hausdorff, (iii) $A \cong A \otimes \mathcal{Q}$, where $\mathcal{Q}$ is the universal UHF algebra, or (iv) $A \cong B \rtimes \mathbb{N}$ where B is an AF-algebra.

Eusebio Gardella, University of Münster
The Rokhlin dimension of strongly outer actions
24 November, 11:00-12:00

The notion of Rokhlin dimension was introduced by Hirshberg, Winter and Zacharias, as a weakening of the Rokhlin property which is strong enough to obtain satisfactory results about the structure of the crossed product. Actions with finite Rokhlin dimension are very common: for instance, Rokhlin dimension at most one is generic for actions of certain discrete amenable groups on Jiang-Su stable C*-algebras. Recently, Liao has shown that for nice enough C*-algebras (unital, separable, simple, with finite nuclear dimension and for which the trace space is a Bauer simplex with fintie dimensional extreme boundary), actions of $\mathbb{Z}^n$ which fix all traces have finite Rokhlin dimension if and only if they are strongly outer. In this talk, we will see that a similar result is valid for finite groups, without any assumptions on the induced action on the

trace space. More explicitly, for finite group actions on C*-algebras as above, strong outerness, Rokhlin dimension at most one, and the weak tracial Rokhlin property, are all equivalent. Furthermore, if a given action satisfies these properties, then it absorbs tensorially the identity on the Jiang-Su algebra.

In fact, as we will discuss, the same strategy can be used to extend this result to a large class of discrete amenable group actions whose induced action on the trace space factors through a finite quotient.

This is joint work with Ilan Hirshberg.

Elizabeth Gillaspy, University of Münster
Twists over étale groupoids and twisted vector bundles
25 November, 11:00-12:00

Given a twist over an étale groupoid, one can construct an associated C^*-algebra which carries a good deal of geometric and physical meaning; for example, the K-theory group of this C^*-algebra classifies D-brane charges in string theory. Twisted vector bundles, when they exist, give rise to particularly important elements in this K-theory group. In this talk, we will present joint work with C. Farsi in which we use the classifying space of the étale groupoid to construct twisted vector bundles, under some mild hypotheses on the twist and the classifying space.

Guihua Gong, University of Puerto Rico
Classification of inductive limit C*-algebras with ideal property
23 November, 14:00-15:00

The class of C*-algebras with the ideal property unifies and generalizes two important class of C*-algebras: the class of real rank zero C*algebras and the class of unital simple C*algebras. In this talk, we will present a classification theorem for AH algebras of no dimension growth with the ideal property which unifies and generalizes the corresponding classifications of real rank zero such algebras due to Dadarlat-Gong and of simple such C*algebras due to Elliott-Gong-Li. One new ingredient in the invariants is the compatibility of Hausdorffized algebraic K_1 group of the cut down algebras which can not be recovered from any old invariant. The talk will be based on two joint papers of Gong-Jiang-Li-Pasnicu and Gong-Jiang-Li.

Hung-Chang Liao, University of Münster
Rokhlin type properties and structure of crossed products
22 November, 16:45-17:45

In view of the recent breakthrough in the classification of simple nuclear C*-algebras, an interesting and challenging question is to understand when a crossed product belongs to the classfiable class. Group actions with Rokhlin type properties tend to pass structual properties from the underlying algebra to the crossed product, and therefore provide partial answers to this question. In this talk we first review the Rokhlin property in dynamics and von Neumann algebras, and then discuss higher dimensional Rokhlin properties in the context of C*-algebras.

Huaxin Lin, University of Oregon
Classification of stably projectionless simple C*-algebras with finite nuclear dimension
24 November, 09:30-10:30

We will report our recent results for the classification of stably projectionless simple C*-algebras with finite nuclear dimension. These include some uniqueness theorems, existence theorems as well as isomorphism theorems (using modified Elliott invariant).

This is a joint work with Guihua Gong.

Zhuang Niu, University of Wyoming
On the classification of simple C*-algebras with trivial K-theory
25 November, 09:30-10:30

Consider the class of simple separable finite C*-algebras which have finite nuclear dimension and are KK-equivalent to the zero algebra. Then this class of C*-algebras is classified by the invariant $(\mathrm{T}^+A, \Sigma A)$, where T^+A is the cone of densely defined lower semicontinuous traces of A with the topology of pointwise convergence, and ΣA is the compact subset consisting of the traces with norm at most one. This is a joint work with G. Elliott.

N. Christopher Phillips, University of Oregon
Mean dimension and radius of comparison
24 November, 14:00-15:00

Let Γ be a countable amenable group, and let X be a compact metric space with a free minimal action of Γ. Then $C^*(\Gamma, X)$ is a stably finite simple separable nuclear C*-algebra satisfying the Universal Coefficient Theorem, but it need not be classifiable in the sense of the Elliott classification program. Based on thin evidence, we hope that the radius of comparison $\mathrm{rc}\big(C^*(\Gamma, X)\big)$ (a measure of the failure of the algebra to have strict comparison of positive elements) should be equal to $\frac{1}{2}\mathrm{mdim}(\Gamma, X)$ (the mean dimension of the action, an invariant from dynamics). The classifiable case corresponds to $\mathrm{rc}(A) = 0$ (conjecturally, but known in many cases).

In this talk, we will discuss parts of the following results:

- For $\Gamma = \mathbf{Z}$ and if X has infinitely many connected components, we have $\mathrm{rc}\big(C^*(\mathbf{Z}, X)\big) \leq \frac{1}{2}\mathrm{mdim}(\mathbf{Z}, X)$. (With Hines and Toms.)
- For $\Gamma = \mathbf{Z}$ and with no additional hypotheses on X, $\mathrm{rc}\big(C^*(\mathbf{Z}, X)\big) \leq 1 + 2\,\mathrm{mdim}(\mathbf{Z}, X)$.
- For $\Gamma = \mathbf{Z}^d$ and if the action has a factor system which is a free minimal action on the Cantor set, $\mathrm{rc}\big(C^*(\mathbf{Z}^d, X)\big) \leq \frac{1}{2}\mathrm{mdim}(\mathbf{Z}^d, X)$.

(Not all details of the last one are yet written.)

The following other results are known, but I won't have time for them in the talk:

- For a class of actions of $\mathbf{Z}$ slightly generalizing the Giol-Kerr examples, and thus including actions with arbitrarily large mean dimension, $\mathrm{rc}\big(C^*(\mathbf{Z}, X)\big) = \frac{1}{2}\mathrm{mdim}(\mathbf{Z}, X)$. (With Hines and Toms.)
- For $\Gamma = \mathbf{Z}$ and with no additional hypotheses on X, if $\mathrm{mdim}(\mathbf{Z}, X) = 0$ then $\mathrm{rc}\big(C^*(\mathbf{Z}, X)\big) = 0$. (Elliott and Niu.)

Mikael Rørdam, University of Copenhagen
A new look on C*-simplicity and the unique trace property
22 November, 09:30-10:30

This is a report on work done by Uffe Haagerup in the spring of 2015 which gives a new way of looking at the recent results by Breuillard, Kalantar, Kennedy and Ozawa on when the reduced group C*-algebra is simple, respectively, has the unique trace property.

Luis Santiago, Lakehead University
Classification of actions of cyclic groups on AI-algebras
24 November, 16:45-17:45

In this talk I will discuss a Cuntz semigroup classification for C*-dynamical systems of the form $\varinjlim(A_n, \alpha_n, G)$, where A_n is an interval algebra and G is a cyclic group of prime order. This is a joint work with Andrew Dean.

Karen Strung, Institute of Mathematics of the Polish Academy of Sciences
Group actions on Smale spaces and associated C*-algebras
23 November, 09:30-10:30

A Smale space is a type of hyperbolic topological dynamical system (X, ϕ) with a particularly tractable local structure. By associating groupoids coming from the contracting, expanding, and asymptotic behaviour of the Smale space, one may construct three C*-algebras. If there is a group action on X that commutes with ϕ, this induces an action on each of these C*-algebras, which in turn allows one to construct crosseed products C*-algebras. I will discuss my recent work with Robin Deeley which studies properties of groups actions on X that allow us to deduce structural properties of the resulting crossed products.

Gábor Szabó, University of Aberdeen
Ocneanu-type uniqueness for certain group actions on strongly self-absorbing C^*-algebras
25 November, 14:00-15:00

The classification problem for amenable group actions on injective factors emerged as a natural aim after Connes' phenomenal classification results in the 70s, which involved the classification of cyclic group actions on type II-factors. After Jones' breakthrough for finite group actions, it was Ocneanu who settled the type II case for all amenable groups in the 80s, in particular verifying that every amenable group has a unique cocycle conjugacy class for outer actions on the hyperfinite II_1-factor. Viewing strongly self-absorbing C*-algebras as the natural C*-algebraic analogs of the hyperfinite II_1-factor, it is natural to ask whether Ocneanu's result can hold its ground in this setting. That is, for a given amenable group G and a strongly self-absorbing C*-algebra $\mathcal{D}$, is there a unique cocycle conjugacy class for strongly outer G-actions on $\mathcal{D}$? It is by now well-known that such rigid behavior is obstructed by torsion in G, but I will argue why the answer to the aforementioned question could be 'Yes' for torsion-free groups. As of yet, however, this problem remained open even for $\mathbb{Z}^3$-actions on the Jiang-Su algebra.

I will explain how the theory of strongly self-absorbing actions can be employed to settle this problem in the abelian case, and maybe even far beyond that.

Hannes Thiel, University of Münster
Abstract bivariant Cuntz semigroups
23 November, 15:30-16:30

We study the category Cu of (abstract) Cuntz semigroups. The Cuntz semigroup $\mathrm{Cu}(A)$ of a C*-algebra A is an object in Cu. This assignment is a continuous functor from C*-algebras to Cu.

In previous work by Antoine, Perera and the speaker, it was shown that there is a natural notion of tensor product in Cu, which gives Cu the structure of a symmetric monoidal category. In some cases, the Cuntz semigroup of a tensor product of C*-algebras can be computed as the tensor product of the Cuntz semigroups of the C*-algebras.

We will show that for any two semigroups S and T in Cu, there is another semigroup $[[S,T]]$ in Cu which plays the role of the morphisms from S to T. We consider $[[S,T]]$ as an *abstract bivariant Cuntz semigroup*. This construction is adjoint to the tensor product in the following sense: For any three abstract Cuntz semigroups S, T and P, there is a natural bijection

$$\mathrm{Hom}_{\mathrm{Cu}}(S \otimes_{\mathrm{Cu}} T, P) \cong \mathrm{Hom}_{\mathrm{Cu}}(S, [[T,R]]).$$

It follows that Cu is a closed monoidal category.

Given C*-algebras A and B, we propose that $[[\mathrm{Cu}(A), \mathrm{Cu}(B)]]$ should be considered as the target of a possible analogue of the universal coefficient theorem for Cuntz semigroups.

(Joint work with Ramon Antoine and Francesc Perera.)

Aaron Tikuisis, University of Aberdeen
The Dixmier property
22 November, 11:00-12:00

The Dixmier property is a property for C*-algebras, originating in '49 in Dixmier's proof of unique trace for II_1 factors, and also featuring prominently in C*-simplicity arguments beginning with Powers in '75. For a unital C*-algebra A, it says that for every element a, one can approximate a central element by averages of unitary conjugates of A.

This property puts some well-known and straightforward restrictions on the tracial and ideal structure of the C*-algebra. For example, a simple C*-algebra with the Dixmier property has at most one tracial state. The converse is a result of Haagerup and Zsidó. I will discuss a new generalization, characterizing the Dixmier property in terms of ideal space and tracial restrictions. I will also talk about a uniform version of the Dixmier property. This is joint work with R. Archbold, and L. Robert.

Andrew Toms, Purdue University
Unitary orbits of self-adjoint operators in C*-algebras
23 November, 16:45-17:45

From as early as the work of Weyl it has been known that the distance between unitary orbits of self-adjoint operators is connected essentially to the spectrum of the operators in question. In this talk we will trace the history of this idea, and present what is quite probably the outer limit of its usefulness among simple separable C*-algebras.

Stuart White, University of Glasgow
2-Isomorphic C*-algebras
23 November, 11:00-12:00

Recent years have seen 'finitely coloured' or 'dimensional' versions of von Neumann properties play important roles in the structure and classification of C*-algebras. In this talk, I'll discuss this viewpoint, and develop the concept of a finitely coloured isomorphism between C*-algebras. This is a more flexible relation between C*-algebras which, for example sees traces but not K-theory. In particular all Kirchberg algebras are two coloured isomorphic.

This is joint work with Jorge Castillejos and Aaron Tikuisis.

Jianchao Wu, Penn State University
Rokhlin dimension for flows
24 November, 15:30-16:30

We introduce a notion of Rokhlin dimension for one-parameter automorphism groups of C^*-algebras, which generalizes Kishimoto's Rokhlin property for flows in the same way Rokhlin dimension for actions of the integers generalizes the corresponding Rokhlin property. The finiteness of this dimension is strong enough to imply a plethora of structural and permanence results on crossed products by such flows, while also general enough to cover all actions arising from free flows on locally compact metrizable spaces with finite covering dimension. In particular, these crossed products have finite nuclear dimension and provide a rich source of classifiable non-unital C^*-algebras. This is joint work with Ilan Hirshberg, Gabor Szabo, and Wilhelm Winter.