Mean dimension and radius of comparison

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Let Γ be a countable amenable group, and let X be a compact metric space with a free minimal action of Γ. Then $C^*(\Gamma, X)$ is a stably finite simple separable nuclear C*-algebra satisfying the Universal Coefficient Theorem, but it need not be classifiable in the sense of the Elliott classification program. Based on thin evidence, we hope that the radius of comparison $\mathrm{rc}(C^*(\Gamma, X))$ (a measure of the failure of the algebra to have strict comparison of positive elements) should be equal to $\frac{1}{2}\mathrm{mdim}(\Gamma, X)$ (the mean dimension of the action, an invariant from dynamics). The classifiable case corresponds to $\mathrm{rc}(A) = 0$ (conjecturally, but known in many cases).

In this talk, we will discuss parts of the following results:

- For $\Gamma = \mathbf{Z}$ and if X has infinitely many connected components, we have $\mathrm{rc}(C^*(\mathbf{Z}, X)) \leq \frac{1}{2}\mathrm{mdim}(\mathbf{Z}, X)$. (With Hines and Toms.)
- For $\Gamma = \mathbf{Z}$ and with no additional hypotheses on X, $\mathrm{rc}(C^*(\mathbf{Z}, X)) \leq 1 + 2\,\mathrm{mdim}(\mathbf{Z}, X)$.
- For $\Gamma = \mathbf{Z}^d$ and if the action has a factor system which is a free minimal action on the Cantor set, $\mathrm{rc}(C^*(\mathbf{Z}^d, X)) \leq \frac{1}{2}\mathrm{mdim}(\mathbf{Z}^d, X)$.

(Not all details of the last one are yet written.)

The following other results are known, but I won't have time for them in the talk:

- For a class of actions of $\mathbf{Z}$ slightly generalizing the Giol-Kerr examples, and thus including actions with arbitrarily large mean dimension, $\mathrm{rc}(C^*(\mathbf{Z}, X)) = \frac{1}{2}\mathrm{mdim}(\mathbf{Z}, X)$. (With Hines and Toms.)

- For $\Gamma = \mathbf{Z}$ and with no additional hypotheses on X, if $\mathrm{mdim}(\mathbf{Z}, X) = 0$ then $\mathrm{rc}(C^*(\mathbf{Z}, X)) = 0$. (Elliott and Niu.)