Cyclic homology

Titles and abstracts

17–21 October 2016

Abhishek Banerjee, Indian Institute of Science
Localization of monoid objects and Hochschild homology
18 October, 15:30-16:30

Given a monoid object A in a symmetric monoidal category $(\mathcal{C}, \otimes, 1)$, we associate a commutative monoid

$$z(A) = Hom_{A-Bimod}(A, A),$$

which we refer to as the set of central elements of A. If A has a centre $Z(A)$ in $\mathcal{C}$, we describe an isomorphism

$$z(A) \cong Hom_{Z(A)-Mod}(Z(A), Z(A)).$$

When $\mathcal{C}$ is closed and complete, we show that every subset

$$S \subseteq Hom_{A-Mod}(A, A)$$

has a centralizer $Z_A(S)$ which is a $Z(A)$-algebra and under certain conditions, we show that

$$Z_A(Hom_{A-Mod}(A, A)) = Z(A).$$

Further, for any multiplicatively closed set $S \subseteq z(A)$, we define the localization M_S of any right A-module M with respect to S and show that several properties of classical localization extend to this context. Finally, under certain conditions, we prove the following theorem: if P is an A-bimodule such that P is symmetric as a bimodule over the center $Z(A)$ of A, we have isomorphisms

$$HH_*(A, P)_S \cong HH_*(A, P_S) \cong HH_*(A_S, P_S)$$

of Hochschild homology groups.

Guillermo Cortiñas, Universidad de Buenos Aires
Cyclic homology and rigid cohomology
18 October, 11:00-12:00

This is a report on joint work with Joachim Cuntz, Ralf Meyer and Georg Tamme. Let V be a complete discrete valuation ring with residue field k. We develop a version of periodic cyclic homology for bornological V-algebras that is defined for not necessarily commutative algebras. For a commutative k-algebra, it recovers Berthelot's rigid cohomology.

Joachim Cuntz, Münster University
Witt vectors and cyclic homology
18 October, 09:30-10:30

We describe a construction of the classical Witt ring that can be used for a natural definition of cyclic homology for commutative algebras over a finite field of positive characteristic.

Vasily Dolgushev, Temple University
Kontsevich's formality quasi-isomorphism is "demystified"
19 October, 11:00-12:00

Back in 1997, Maxim Kontsevich proved that the algebra of polydifferential operators on a smooth manifold is formal (i.e. quasi-isomorphic to its cohomology). The known constructions of Kontsevich's formality quasi-isomorphism involve "transcendental" tools: the original construction, due to Kontsevich, is based on the configuration space integral, while Tamarkin's construction involves Drinfeld's associator. In my talk, I will describe an explicit recursive construction whose output is a formality quasi-isomorphism for polydifferential operators defined over rationals. My talk is based on paper http://arxiv.org/abs/1306.6733

Alexander Gorokhovsky, University of Colorado, Boulder
Primary and secondary invariants of pseudodifferential symbols
21 October, 09:30-10:30

We compare different constructions of cyclic cocycles for the algebra of complete symbols of pseudodifferential operators and show that our comparison result leads to interesting index-theoretic consequences and a construction of invariants of the algebraic K-theory of the algebra of pseudodifferential symbols.

Lars Hesselholt, Nagoya University
and University of Copenhagen
Topological Hochschild homology and the Hasse-Weil zeta function
19 October, 09:30-10:30

In the nineties, Deninger gave a detailed description of a conjectural cohomological interpretation of the (completed) Hasse-Weil zeta function of a regular scheme proper over the ring of rational integers. He envisioned the cohomology theory to take values in countably infinite dimensional complex vector spaces and the zeta function to emerge as the regularized determinant of the infinitesimal generator of a Frobenius flow. In this talk, I will explain that for a scheme smooth and proper over a finite field, the desired cohomology theory naturally appears from the Tate cohomology of the action by the circle group on the topological Hochschild homology of the scheme in question.

Dmitry Kaledin, Steklov Math Institute, Moscow
Co-periodic cyclic homology
19 October, 14:00-15:00

Periodic cyclic homology is defined by taking the product-total complex of a certain bicomplex. For algebras over $\mathbb{Q}$, taking the sum-total complex of the same bicomplex gives 0. It has been suggested by Kontsevich some years ago that in char p, the sum-total complex is a non-trivial and interesting invariant. At the time, the suggestion was not pursued seriously; however, recently a very similar phenomenon appeared in the work of Beilinson and Bhatt on p-adic Hodge theory. I want to revisit the subject and follow through on Kontsevich's idea, both for algebras and DG algebras. The resulting "co-periodic cyclic homology" theory is derived

Morita-invariant, is an algebra over Laurent polynomials in one variable of cohomological degree -2, and exhibits several other interesting features.

Masoud Khalkhali, University of Western Ontario
Monoidal categories, 2-traces, and cyclic cohomology
20 October, 09:30-10:30

This talk is an introduction to our joint paper (with M. Hassanzadeh and I. Shapiro) where we extend the formalism of Hopf cyclic cohomology to a monoidal category context. As a byproduct we derive all existing types of Hopf cyclic theories from a unified formalism.

Niels Kowalzig, Universitá di Roma La Sapienza
When Ext is a Batalin-Vilkovisky algebra
19 October, 17:00-18:00

We show under what conditions the complex computing general Ext-groups carries the structure of a cyclic operad such that Ext becomes a Batalin-Vilkovisky algebra. This is achieved by transferring cyclic cohomology theories for the dual of a (left) Hopf algebroid to the complex in question, which asks for the notion of contramodules introduced along with comodules by Eilenberg-Moore half a century ago. Another crucial ingredient is an explicit formula for the inverse of the Hopf-Galois map on the dual, by which we illustrate recent categorical results and answering a long-standing open question. As an application, we prove that the Hochschild cohomology of an associative algebra A is Batalin-Vilkovisky if A itself is a contramodule over its enveloping algebra $A \otimes A^{\mathrm{op}}$. This is, for example, the case for symmetric algebras and Frobenius algebras with semisimple Nakayama automorphism. We also recover the construction for Hopf algebras.

Ulrich Kraehmer, University of Glasgow
What if $bB + Bb \neq 0$?
20 October, 15:30-16:30

One encounters this question in many different contexts: noncommutative De Rham theory, twisted cyclic homology, Hopf-cyclic homology, or in the Dwyer-Kan correspondence. Cuntz and Quillen gave beautiful answers for the example of noncommutative De Rham theory, and our aim in this talk is to see whether they also work in greater generality. Based on joint work with Dylan Madden.

Ryszard Nest, University of Copehagen
On analytic constructions of group cocycles
17 October, 10:30-12:00

One of the most important group cocycles is the two-cocycle giving the central extension of the restricted general linear group of a polarised Hilbert space (H, H_+). It has a wide range of applications, ranging from the conformal field theory to invariants of the algebraic K-theory. It can be seen as a two-cocycle associated to the action of the group $GL_{res}(H, H_+)$ on the category of idempotents $P \in \mathcal{B}(H)$ such that $[P_{H_+}, P] \in \mathcal{L}^2(H)$. In another disguise it is given by the Connes-Karoubi multiplicative character on the universal two-summable Fredholm module.

More generally, given an action of a group G on an n-category satisfying certain conditions, one can construct a (n+1)-cocycle on G. A well known example is the n-Tate space, essentially an algebra of the form $K = k((s_1))((s_2))\dots((s_n))$, where the group is the group of invertibles in K and the n-category structure comes from the natural filtration of K.

The corresponding cocycles, when evaluated on $K^{alg}_{n+1}(K)$, reproduce the Tate tame symbol. However, the constructions are purely algebraic and do not seem to extend to the analytic context, as in the case of $n = 1$.

In this talk we will sketch a construction of a family of two-categories associated to a pair of commuting idempotents P and Q on a Hilbert space and construct the associated three cocycle on the associated groups. For example, in the case of a two-Tate space, this produces an extension of the Tate symbol and the corresponding invariant of K^{alg}_3 from the 2-Tate space to $C^\infty(\mathrm{T}^2)$. As another example we get a corresponding invariant of K^{alg}_3 of the non-commutative torus $C^\infty(\mathrm{T}^2_\theta)$.

The construction is based on the properties of the determinant of Fredholm operators, in particular on the existence of the canonical perturbation isomorphism $Det(T) \simeq Det(S)$ associated to a pair of Fredholm operators T and S satisfying $T - S \in \mathcal{L}^1(H)$.

This is a joint work with Jens Kaad and Jesse Wolfson.

Markus Pflaum, University of Colorado, Boulder
On the Hochschild homology of the convolution algebra of a proper Lie groupoid
21 October, 15:30-16:30

In his work on the cyclic homology theory of the convolution algebra of a compact Lie group action on a manifold Brylinski introduced a complex of so-called basic relative forms. We extend that construction to the case of arbitrary proper Lie groupoids and interpret it as a particular complex of Grauert- Grothendieck forms on the inertia space of the Lie groupoid. We then show how the complex of basic relative forms on the inertia space relates to the Hochschild homology theory of the convolution algebra of the Lie groupoid.

Raphaël Ponge, Seoul National University
Cyclic Homology of Crossed-Product Algebras
21 October, 14:00-15:00

This talk will be a preliminary report on an explicit computation of the cyclic homology and periodic cyclic homology of crossed-product algebras over commutative rings. By explicit computation it is meant the construction of explicit quasi-isomorphisms. This enables us to recover and somewhat simplify various known results by Baum-Connes, Brylinski-Nistor, Crainic, Feigin-Tsygan, Getzler-Jones, and Nistor, among others. In particular, we recover the spectral sequences of Feigin-Tsygan and Getzler-Jones, and obtain several other spectral sequences as well. The approach is purely algebraic. It grew out of an attempt to extend to crossed-product algebras the algebraic approach of Marciniak to the computation of the cyclic homology of group rings by Burghelea. At the conceptual level, we introduce a generalization of the cyclindrical complexes of Getzler-Jones. This provides us with the relevant homological tool to understand the cyclic homology of crossed-product algebras, especially in the non-torsion case.

Michael Pushnigg, Institut de Mathématiques de Marseille
Some calculations of local cyclic cohomology groups
18 October, 17:00-18:00

We outline the calculation of bivariant local cyclic cohomology for group Banach algebras of hyperbolic groups and of groups acting properly, isometrically and cocompactly on a CAT(0)-space. While the cyclic cohomology of group rings in general is well known since a long time, our results in the case of the group Banach algebras is new and requires a quite detailed knowledge of the groups under study. This is partly joint work with Benjamin Andrysiak.

Ilya Shapiro, University of Windsor
Some invariance properties of cyclic cohomology with coefficients
20 October, 11:00-12:00

We will discuss a definition of cyclic cohomology with coefficients in the setting of abelian monoidal categories (such as the motivating example of modules or comodules over a Hopf algebra) and its invariance properties. This setting is appropriate for generalizing Hopf cyclic cohomology with coefficients in a stable anti-Yetter-Drinfeld module and furthermore sheds light on some properties of Hopf cyclic cohomology itself.

Georgy Sharygin, Moscow State University
Vector fields and deformation quantization
21 October, 11:00-12:00

The purpose of this simple talk is to give few examples of questions and constructions, arising from a (rather naive) attempt to transfer some algebraic structures on a Poisson manifold (M, π) to the quantized algebra of its functions.

We begin with the case of a (finite-dimensional) Lie algebra action on M by Poisson vector fields; as one knows, every Poisson field gives rise (via the formality morphism) to a differentiation of the deformed algebra. However, this construction does not necessarily map commutators into commutators, the difference being given by an inner derivative. This gives a sequence of obstructions in a Chevalley cohomology of the Lie algebra with coefficients in Lichnerowicz-Poisson complex. Another interesting example, which I shall talk about are the so called Nijenhuis vector fields, which play an important role in the argument shift method and its generalizations. In this case we obtain obstructions in a twisted version of Hochschild cohomology.

Serkan Sutlu, Işik University
A characteristic map for compact quantum groups
20 October, 14:00-15:00

If G is a compact Lie group and g is its Lie algebra, then we construct a map from the Hopf-cyclic cohomology of the quantum enveloping algebra $U_q(g)$ to the twisted cyclic cohomology of quantum group algebra $O(G_q)$. We show that this characteristic map is nontrivial, and captures the Schmudgen-Wagner index cocycle, associated with the volume form of the differential calculus on the standard Podleś sphere, in the image. (Joint work with A. Kaygun)

Georg Tamme, Universität Regensburg
K-theory of non-archimedean Banach algebras
18 October, 14:00-15:00

This is a report on joint work with Moritz Kerz and Shuji Saito. I will motivate and discuss an analogue of topological K-theory for non-archimedean Banach algebras. In particular, I will explain what we know about the comparison with algebraic K-theory and mention possible relations with cyclic homology.

Boris Tsygan, Northwestern University
Two-categories with a trace functor and cyclc homology
19 October, 15:30-16:30

It is well known that associative algebras form a two-category where objects are algebras, one-morphisms are bimodules, and two-morphisms are bimodule morphisms. This two-category admits a trace functor which assigns to a bimodule over a ring its zero Hochschild homology. (The notion of a two-category with a trace functor is a categorification of the notion of an algebra with a trace; it had been used by various authors, in particular Kaledin and Tamarkin). We construct the derived version of the above, i.e. the structure of a homotopy two-category with a trace functor whose objects are algebras, one-morphisms are morphism of algebras (each defines a bimodule), Hochschild cochain complexes, and where the trace functor assigns to an algebra the Hochschild chain complex. We explain how this implies other structures on Hochschild (co)chains, starting with the cyclic differential B. The statements regarding cochains are our version of the theorems of McClure-Smith, Batanin, Tamarkin, and Lurie. This is a joint work with Rebecca Wei.

Hang Wang, The University of Adelaide
Higher Donaldson invariants
17 October, 14:00-15:30

Fundamental groups and "higher theory" are useful tools in the study of geometry and topology of compact Riemannian manifolds. They give rise to refined invariants in K-theory of a group C*-algebra, and can be calculated by pairing their Chern character in cyclic homology with group cocycles. Motivating examples include the Connes-Moscovici localised index formulas and Lott's construction of higher eta-invariants in cyclic homology. The former are used to prove the Novikov conjecture for hyperbolic groups, and the latter give rise to refined spectral invariants for compact manifolds with boundary. Donaldson invariants, an important concept in gauge theory of closed 4-manifolds, also have a counterpart in the higher theory. We will show that they can be defined and calculated using cyclic homology. This is work in progress with T. Kato and H. Sasahira.