On analytic constructions of group cocycles

Ryszard Nest, University of Copehagen

17 October, 10:30-12:00

One of the most important group cocycles is the two-cocycle giving the central extension of the restricted general linear group of a polarised Hilbert space (H, H_+). It has a wide range of applications, ranging from the conformal field theory to invariants of the algebraic K-theory. It can be seen as a two-cocycle associated to the action of the group $GL_{res}(H, H_+)$ on the category of idempotents $P \in \mathcal{B}(H)$ such that $[P_{H_+}, P] \in \mathcal{L}^2(H)$. In another disguise it is given by the Connes-Karoubi multiplicative character on the universal two-summable Fredholm module.

More generally, given an action of a group G on an n-category satisfying certain conditions, one can construct a (n+1)-cocycle on G. A well known example is the n-Tate space, essentially an algebra of the form $K = k((s_1))((s_2))\dots((s_n))$, where the group is the group of invertibles in K and the n-category structure comes from the natural filtration of K.

The corresponding cocycles, when evaluated on $K^{alg}_{n+1}(K)$, reproduce the Tate tame symbol. However, the constructions are purely algebraic and do not seem to extend to the analytic context, as in the case of $n = 1$.

In this talk we will sketch a construction of a family of two-categories associated to a pair of commuting idempotents P and Q on a Hilbert space and construct the associated three cocycle on the associated groups. For example, in the case of a two-Tate space, this produces an extension of the Tate symbol and the corresponding invariant of K_3^{alg} from the 2-Tate space to $C^\infty(\mathrm{T}^2)$. As another example we get a corresponding invariant of K_3^{alg} of the non-commutative torus $C^\infty(\mathrm{T}^2_\theta)$.

The construction is based on the properties of the determinant of Fredholm operators, in particular on the existence of the canonical perturbation isomorphism $Det(T) \simeq Det(S)$ associated to a pair of Fredholm operators T and S satisfying $T - S \in \mathcal{L}^1(H)$.

This is a joint work with Jens Kaad and Jesse Wolfson.