Harmonic Analysis, Complex Analysis, Spectral Theory and all that

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Abstracts of talks

Nikolai Nikolski (Université de Bordeaux),
Stefanie Petermichl (Université Paul Sabatier),
Sergei Treil (Brown Univeristy)

Opening lecture

Tomasz Adamowicz (Institute of Mathematics of the Polish Academy of Sciences)

Prime ends and quasiconformal mappings

(joint work with Ben Warhurst)

The studies of prime ends have long history involving various approaches, for example due to Carathéodory, Näkki, Väisälä and Zorich. In the talk we present some of those theories, including recent developments in metric measure spaces and, in particular, in the setting of Heisenberg groups. Moreover, we discuss problems of continuous and homeomorphic extensions of mappings to the topological and prime end boundaries in the Euclidean setting and the setting of metric measure spaces for a class of mappings generalizing quasiconformal mappings. The presentation also includes some results on the boundary behavior of quasiconformal mappings between domains in Heisenberg groups.

Kari Astala (University of Helsinki)

Invertibility of Beltrami operators, Burkholder functional and Chord-arc curves

Many of the subtle L^p-properties related to the Beurling transform still remain much of a mystery. There are many different ways to approach the operator, notably for instance the Bellman function method developed by Sasha Volberg and his collaborators. The quasiconformal method provides another route, and in this talk I wish to give an overview of what one can achieve via this point of view.

Alexander Austin (University of California, Los Angeles)

Logarithmic potentials and quasiconformal flows on the Heisenberg group

We show that if the total variation of the measure associated to a logarithmic potential u on the Heisenberg group ($\mathbb{H}$) is sufficiently small, then there exists a quasiconformal mapping f of $\mathbb{H}$ such that the Jacobian of f is almost everywhere comparable to e^{2u}. This is exactly analogous to a theorem of Bonk, Heinonen, and Saksman in the Euclidean case. It is a first look at the quasiconformal Jacobian problem outside the Euclidean setting, and is a testament to the richness of the family of quasiconformal mappings of $\mathbb{H}$. As a precursor, we extend the flow method of Korányi and Reimann for generating quasiconformal mappings of $\mathbb{H}$, putting it on roughly the same footing as that of $\mathbb{R}^n$, for $n \geq 3$.

Michael Benedicks (KTH Royal Institute of Technology)

Mathematical problems in density functional theory and unique continuation

(joint work with André Laestadius)

Density functional theory is a computational quantum mechanical modelling method used in physics, chemistry and materials science to investigate the electronic structure (principally the ground state) of many-body systems, in particular atoms, molecules, and the condensed phases. A central theoretical part are the theorems formulated by Hohenberg and Kohn (Nobel prize in chemistry 1998) concerning an equivalence between variational problems and the quantum mechanical many body problems. These results were put on a rigourous mathematical basis by Elliot Lieb, but the case with exterior magnetic fields were not treated until recently and mathematical problems remain. An important aspect occurring is the theory of unique continuation for PDE:s.

Roman Bessonov (Saint Petersburg State University)

Krein strings via truncated Toeplitz operators

I will present an approach to inverse spectral problems for Krein string equation based on application of the theory of Toeplitz operators on the Paley–Wiener space. The main result says that if a spectral measure of a Krein system is absolutely continuous with density separated from zero and infinity, then the coefficient of the system is the derivative of a BMO function on the positive half-axis. The result has straightforward applications to the theory of Krein's orthogonal entire functions and the problem of triangular factorization for positive operators. These applications will be discussed at the the beginning of the talk; they do not require any knowledge of spectral theory.

Nicholas Boros (Olivet Nazarene University)

Matrix weights, Littlewood Paley inequalities and the Riesz transform

(joint work with Nikolaos Pattakos)

We will discuss weighted estimates for the squares of the Riesz transforms $R_1^2, \ldots, R_m^2$ on $L^2(W)$ where $W \in \mathbb{C}^{d\times d}$ is an A_2 weight. We will show that if the "Heat A_2 characteristic" of W is sufficiently close to 1 then there is a dimensional constant $c > 0$ such that

$$\|R_i^2\|_{2,W} \leq 1 + c\sqrt{[W]_{A_2^h} - 1},$$

for all $i = 1, \ldots, m$. This is accomplished by proving a Littlewood–Paley estimate with the use of the Bellman function technique.

Michael Christ (University of California, Berkeley)

A sharpened Riesz–Sobolev inequality

The Riesz-Sobolev inequality provides an upper bound, in integral form, for the convolution of indicator functions of subsets of Euclidean space. A theorem of Burchard characterizes cases of equality. We report on a sharper form of the inequality, which gives a quantitative form of Burchard's theorem. If time permits, another application of the method will be sketched.

Krzysztof Ciosmak (Institute of Mathematics of the Polish Academy of Sciences)

Differentiable functions on modules and the equation $\mathrm{grad}(w) = M\,\mathrm{grad}(v)$

Let A be a finite-dimensional, commutative algebra over $\mathbb{R}$ or $\mathbb{C}$ and let B be a finitely generated A-module. We define A-differentiable functions on module B – these are differentiable functions such that the derivative at any point is A-linear.

Let U be an open, bounded and convex subset of B. When A is singly generated and B is arbitrary or A is arbitrary and B is a free module, we give an explicit formula for an A-differentiable functions on U of the prescribed class of differentiability in terms of real and complex differentiable functions. It appears, even in the case of real algebras, that certain components of A-differentiable function are of higher differentiability than the function itself.

Using the formula for A-differentiable functions on modules we find the complete solution of the equation $\mathrm{grad}(w) = M\,\mathrm{grad}(v)$.

Luigi D'Onofrio (Università degli Studi di Napoli Parthenope)

Note on Lusin (N) *condition and the distributional determinant*

(joint work with Stanislav Hencl, Jan Maly and Roberta Schiattarella)

I will present a joint work in which we show that for a continuous mapping, the validity of the Lusin (N) condition implies that the distributional Jacobian equals to the pointwise Jacobian.

Guy David (Université Paris-Sud)

Boundary regularity for minimal sets of dimension 2

We try to describe regularity result at the boundary, for two-dimensional minimal sets that satisfy a sliding Plateau condition. Here the boundary is a smooth curve. We only give a partial description, that can be seen as a list of singularities, modulo C^1 diffeomorphisms (work in progress).

Oliver Dragičević (University of Ljubljana)

Convexity of power functions and applications to functional calculus

(joint work with Andrea Carbonaro)

We prove that every generator of a symmetric contraction semigroup on a σ-finite measure space admits, for $1 < p < \infty$, a Hörmander-type holomorphic functional calculus on L^p in the sector of angle $\arcsin|1 - 2/p|$. The obtained angle is optimal.

The proof features a combination of the heat flow and a particular Bellman function. While the core of this approach is not new, the modification suited for the functional calculus is. We extend it to give:

- the optimal version of the functional calculus for nonsymmetric Ornstein-Uhlenbeck operators;

- a rather optimal version of the dimension-free bilinear embedding for divergence-form operators with complex symbols.

A point we make is that our results and methods are related to the contractivity of the underlying semigroups and, on the other hand, to the convexity of power functions of a single complex variable.

Polona Durcik (Universität Bonn)

Quantitative estimates for the simplex Hilbert transform

(joint work with Vjekoslav Kovač and Christoph Thiele)

The simplex Hilbert transform is a multilinear generalization of the triangular Hilbert transform. Its L^p boundedness is an open problem. We present some progress on improving the bound by Zorin–Kranich on the growth of truncations of the simplex Hilbert transform.

Konstantin Dyakonov (ICREA / Universitat de Barcelona)

Locally inner functions and their derivatives

It has been noticed that the derivative of an inner function is never outer, except in trivial cases. Our current purpose is to find out whether (and to what extent) this carries over to more general classes of analytic functions on the disk. Specifically, we are concerned with the case of a "locally inner" function, i.e., a unit-norm H^∞ function whose modulus equals 1 on a part of the boundary.

Benjamin Eichinger (Johannes Kepler University Linz)

Szegö–Widom asymptotics of Chebyshev polynomials on circular arcs

We consider the Chebyshev polynomials T_n on a circular arc A_α, i.e., the monic polynomials of degree at most n that minimizes the sup-norm $\|T_n\|_{A_\alpha}$. Thiran and Detaille found an explicit formula for the asymptotics of $\|T_n\|_{A_\alpha}$. We give the Szegö-Widom asymptotics of the domain explicitly. That is, the limit of the properly normalized extremal functions T_n. Moreover, we solve a similar problem with respect to the upper envelope of a family of polynomials uniformly bounded on A_α. Our computations show that in the proper normalization the limit of the upper envelope is the diagonal of a reproducing kernel of a certain Hilbert space of analytic functions.

Vladimir Eiderman (Indiana University)

Maximum principle for the s-Riesz transform

Let μ be a measure in $\mathbb{R}^d$, and let

$$R^s\mu(x) = \int \frac{y-x}{|y-x|^{s+1}}\, d\mu(y), \quad x, y \in \mathbb{R}^d, \quad 0 < s < d.$$

We consider the following conjecture:

$$\max_{x\in\mathbb{R}^d} |R^s\mu(x)| \le C \max_{x\in\operatorname{supp}\mu} |R^s\mu(x)|, \quad C = C(d,s).$$

This relation is proved for $0 < s < 1$ and $d-1 \le s < d$, and is still an open problem in the general case. The maximum principle is important for the problem on the connection between geometric properties of a measure and boundedness of the Calderón-Zygmund operator in $L^2(\mu)$, and also is of independent interest. We sketch the ideas of proofs (which are completely different) in the two cases mentioned above, and indicate some special classes of measures for which the conjecture is proved for $0 < s < d$.

Alexandre Eremenko (Purdue University)

A remarkable (anti)-holomorphic dynamical system

(joint work with Walter Bergweiler)

The Green function on a flat torus is defined as a periodic solution u of the equation

$$\Delta u = -\delta + 1/|T|,$$

where $|T|$ is the area of the torus. The question is how many critical points (solutions of $\nabla u = 0$) can it have. C.-S. Lin and C.-L. Wang proved in 2010 that the answer is 3 or 5, depending on the parameter of the torus. Their proof is quite complicated; it uses some advanced non-linear PDE techniques.

Searching for a simpler proof of this theorem, we discovered a complex one-parametric family of holomorphic dynamical systems with a remarkable property that the parameter plane consists of just two hyperbolic components and an analytic curve separating them.

Adi Glücksam (Tel Aviv University)

Translation invariant probability measures on the space of entire functions

(joint work with Lev Buhovski, Alexander Logunov and Mikhail Sodin)

Twenty years ago Benjy Weiss constructed a collection of non-trivial translation invariant probability measures on the space of entire functions. In this talk we will present another construction of such a measure, and give upper and lower bounds for the possible growth of entire functions in the support of such a measure. We will also discuss "uniformly recurrent" entire functions, their connection to such constructions, and their possible growth.

Haakan Hedenmalm (KTH Royal Institute of Technology)

Bloch functions and notions of asymptotic variance

Ritva Hurri-Syrjanen (University of Helsinki)

Aspects of local-to-global results

(joint work with Niko Marola and Antti V. Vähäkangas)

In this talk, we discuss results on the function space which is larger than the well known BMO space, and was also introduced by Fritz John and Louis Nirenberg in their paper 'On functions of bounded mean oscillation', 1961. As opposed to BMO functions, which have exponentially decaying distribution functions, a function in this larger space is known to belong to a weak L^p-space; the inclusion being strict. We localize the condition given by John and Nirenberg and prove a local to global result between this localized version and the original one. We also consider necessary and sufficient conditions for the corresponding weak type inequalities.

Tuomas Hytönen (University of Helsinki)

A tale of two theorems ($T1$ *and* $A2$)

T1 theorems are a characterisations of the boundedness of singular integral operators in terms of "testing conditions". The A2 theorem is the sharp norm bound for the same operators on weighted Lebesgue spaces. My aim is to highlight some crossing-points of the two theories; these are also among the main crossing-points of my own research with that of Sasha Volberg.

Benjamin Jaye (Kent State University)

Necessary and sufficient conditions for the boundedness of Calderón–Zygmund operators in terms of non-oscillatory quantities

(joint work with Fedor Nazarov, Maria Carmen Reguera and Xavier Tolsa)

We shall describe several results which characterize measures μ in $\mathbb{R}^d$ for which certain classes of associated singular integral operators are bounded in $L^2(\mu)$.

Agnieszka Kałamajska (University of Warsaw)

Hardy-type inequalities derived from degenerated p-harmonic problems

The purpose of this talk is to show certain new method to construct Hardy type inequalities when knowing the nonnegative solution to the anticoercive partial differential inequality of elliptic type involving degenerated p–Laplacian: $-\Delta_{p,a}u := -\mathrm{div}(a(x)|\nabla u|^{p-2}\nabla u) \geq b(x)\Phi(u)$, where u is defined on a domain Ω. The method leads often to inequalities derived with best constants. The talk will be based on recent paper [1] and earlier contributions [2],[3]:

[1] P. Drabék, A. Kałamajska, I. Skrzypczak, *Caccioppoli–type estimates and Hardy–type inequalities derived from degenerated p–harmonic problems,* preprint 2016.

[2] A. Kałamajska, K. Pietruska-Pałuba, I. Skrzypczak, *Nonexistence results for differential inequalities involving A–Laplacian,* Adv. Diff. Eqs. 17 (3–4) (2012), 307–336.

[3] I. Skrzypczak, *Hardy–type inequalities derived from p–harmonic problems*, Nonlinear Anal. TMA 93 (2013), 30–50.

Anna Kamont (Institute of Mathematics of the Polish Academy of Sciences)

Directional wavelet projections and interpolatory estimates for the Riesz transforms revisited

(joint work with Paul F. X. Müller)

The problem presented in this talk has its origin in papers [4], [2], [3]. In these papers, the authors obtained the following interpolation inequality: let $\Phi \in L^2(\mathbb{R}^d)$ be a fixed element of the Haar system on $\mathbb{R}^d$, or a fixed element of a wavelet system on $\mathbb{R}^d$, satisfying some Hölder condition, and let $P_\Phi u = \sum_{j\in\mathbb{Z},k\in\mathbb{Z}^d}(u,\Phi_{j,k})\Phi_{j,k}$ be the orthogonal projection onto the space spanned by $\{\Phi_{j,k}(\cdot) = 2^{dj/2}\Phi(2^j\cdot -k), j \in \mathbb{Z}, k \in \mathbb{Z}^d\}$. Then the following inequality holds:

$$(*) \qquad \|P_\Phi u\|_p \leq C\|u\|_p^{1-\theta}\|R_i u\|_p^\theta, \quad 1 < p < \infty,$$

where R_i is Riesz transform on $\mathbb{R}^d$ in i-th direction, and $0 < \theta < 1$ is an exponent, depending only on the wavelet under consideration and p. In case of the Haar wavelet, there is $\theta = 1/2$ for $2 \leq p < \infty$ and $\theta = 1 - 1/p$ for $1 < p \leq 2$, and these exponents are best possible. In case of a wavelet satisfying the Hölder condition with an exponent $0 < \alpha < 1$, there is $\theta = \alpha$ for all p; there is also a version of $(*)$ for $\alpha = 1$.

The aim of this talk is to explain the nature of the exponent θ appearing in $(*)$ in case $p = 2$. For $\Phi = \phi \otimes \psi$ with $\phi \in L^2(\mathbb{R}^{d-1})$ and $\psi \in L^2(\mathbb{R})$, and $i = d$ in $(*)$, we obtain a characterization of exponents θ in $(*)$ in terms of coefficients of an expansion of φ with respect to a wavelet basis on $\mathbb{R}^{d-1}$. This in turn allows us to formulate some necessary condition and some sufficient condition for $(*)$ in terms of regularity of φ. In particular, note that regularity of ψ does not matter here.

The talk is based on paper [1], joint with Paul F. X. Müller (J. Kepler University, Linz, Austria).

[1] A. Kamont, P. F. X. Müller, Directional wavelet projections and interpolatory estimates for Riesz transform revisited. In preparation.

[2] J. Lee, P. F. X. Müller, S.Müller, Compensated compactness, separately convex functions and interpolatory estimates between Riesz transforms and Haar projections. Comm. Partial Differential Equations, 36 (4), 547–601, 2011.

[3] P. F. X. Müller, S. Müller, Interpolatory estimates, Riesz Transforms and Wavelet Projections. Revista Math. Iberoam. (2016).

[4] S. Müller, Rank-one convexity implies quasiconvexity on diagonal matrices. Internat. Math. Res. Notices 20, 1087–1095, 1999.

Avner Kiro (Tel Aviv University)

Taylor coefficients of Beurling classes of smooth functions and moment summation methods

The talk is will be devoted to two problems in the theory of Beurling and Carleman classes of smooth functions. The first one is to describe the image of a Beurling or Carleman class under Borel's map $f \mapsto (f^{(n)}(0)/n!)_{n\geq 0}$. The second one is how to construct a function in Beurling or Carleman class with prescribed Taylor coefficients. In the talk, I will present solutions to both problems in some Beurling and Carleman classes.

Sergei Kisliakov (St. Petersburg Department of V. A. Steklov Institute of Mathematics of the Russian Academy of Sciences)

Some observations concerning the corona theorem

This is a short survey of fairly recent results (obtained by the author and D. Rutsky, partly jointly, partly separately) concerning the classical corona theorem in the disk. The relationship between various estimates for solutions of the Besout equation in analytic functions will be discussed, along with certain links with interpolation theory and fixed point theorems.

Vjekoslav Kovač (University of Zagreb)

Bellman functions and L^p estimates for paraproducts

(joint work with Kristina Ana Škreb)

We regard dyadic paraproducts as multilinear forms. Even though they are well-known to satisfy L^p estimates in the whole Banach range of exponents, one might want to give a direct proof or study the behavior of the constants. We find the explicit formulae for the Bellman functions in the spirit of the Bellman function by Nazarov, Treil, and Volberg. Then we apply the same Bellman function in various other settings, such as the martingale paraproducts of Bañuelos and Bennett or the paraproducts with respect to the heat flows.

Sebastian Król (Nicolaus Copernicus University)

Application of weighted inequalities to extrapolation of the maximal regularity of the abstract Cauchy problems

(joint work with Ralph Chill)

We prove weighted estimates for singular integral operators which operate on function spaces on a half-line. The class of admissible weights includes Muckenhoupt weights and weights satisfying Sawyer's one-sided conditions. The kernel of the operator satisfies relaxed Dini conditions. We apply the weighted estimates to extrapolation of maximal L_p regularity of first order, second order and fractional order Cauchy problems into weighted rearrangement invariant Banach function spaces. In particular, we provide extensions, as well a unification of recent results due to Auscher & Axelsson and Chill & Fiorenza.

Bartosz Langowski (Wroclaw University of Technology)

Sobolev and potential spaces related to Jacobi expansions

We define and study Sobolev spaces associated with Jacobi expansions. We prove that these Sobolev spaces are isomorphic, in the Banach space sense, with potential spaces (for the Jacobi 'Laplacian') of the same order. This is an essential generalization and strengthening of the recent results [1] concerning the special case of ultraspherical expansions, where in addition a restriction on the parameter of type was imposed. We also apply a symmetrization procedure to the setting of Jacobi expansions and study potential spaces in the resulting situation.

Moreover, we give a characterization of the Jacobi potential spaces of arbitrary order in terms of suitable fractional square functions. As an auxiliary result of independent interest we prove L^p-boundedness of these fractional square functions.

[1] J. J. Betancor, J. C. Fariña, L. Rodríguez-Mesa, R. Testoni, J. L. Torrea, *A choice of Sobolev spaces associated with ultraspherical expansions*, Publ. Math. 54 (2010), 221–242.

[2] B. Langowski, *Sobolev spaces associated with Jacobi expansions*, J. Math. Anal. Appl. 420 (2014), 1533–1551.

[3] B. Langowski, *On potential spaces related to Jacobi expansions*, J. Math. Anal. Appl. 432 (2015), 374–397.

[4] B. Langowski, *Potential and Sobolev spaces related to symmetrized Jacobi expansions*, Symmetry, Integrability and Geometry: Methods and Applications; SIGMA 11 (2015), 073, 17 pages.

Alexander Logunov (Tel Aviv University)

Nodal sets of Laplace Eigenfunctions: pursuing the conjectures by Yau and Nadirashvili

We will discuss the recent progress in understanding zero sets of harmonic functions and Laplace eigenfunctions.

Eugenia Malinnikova (Norwegian University of Science and Technology)

Ratios of harmonic functions

(joint work with Alexander Logunov)

We study the ratio of harmonic functions u, v which have the same zero set Z in the unit ball B. In dimension two such ratios were considered by Dan Mangoubi. The ratio $f = u/v$ can be extended to a real analytic nowhere vanishing function in B. We prove the Harnack inequality and the gradient estimate for such ratios in any dimension: for a given compact set $K \subset B$ we show that $\sup_K |f| \leq C_1 \inf_K |f|$ and $\sup_K |\nabla f| \leq C_2 \inf_K |f|$, where C_1 and C_2 depend on K and Z only. In dimension two the constants in these inequalities depend only on the number of nodal domains of u, i.e. the number of connected components of $B \setminus Z$.

Paul F. X. Müller (Johannes Kepler University Linz)

Compensated compactness, interpolatory estimates, Riesz transforms and wavelet projections

We discuss how problems from the Calculus of Variations give rise to interpolatory estimates between directional Haar projections and Riesz transforms. We present those estimates and together with generalizations to wavelet projections with emphasis on the sharpness of the interpolatory exponents.

Piotr Nayar (University of Pennsylvania)

Brunn–Minkowski inequality for log-concave measures

(joint work with Galyna Livshyts, Arnaud Marsiglietti and Artem Zvavitch)

We show that the Brunn-Minkowski inequality holds true for general coordinate-wise symmetric log-concave measures and coordinate-wise symmetric convex sets. In particular, we partially solve the Gaussian Brunn-Minkowski conjecture of R. Gardner and A. Zvavitch.

Alon Nishry (University of Michigan)

Gaussian Complex Zeros: Large fluctuations and conditional distribution

(joint work with S. Ghosh)

We study the Gaussian Entire Function, a random Taylor series with independent complex Gaussian coefficients, which is distinguished by the invariance of its zero set with respect to the isometries of the complex plane. We consider very rare events, where the number of zeros in a large disk is far from the expected value. Of particular interest is the 'hole' event, that is, when there are no zeros in a large disk. We find the precise logarithmic asymptotics of the probability of these rare events. In addition, we find the conditional distribution of the zeros, given an event of this type. In particular, we show that, conditioned on the hole event, there is a large gap in the distribution of the zeros. This answers a question posed by Nazarov and Sodin, and is in stark contrast to the corresponding result known to hold in the random matrix setting, where such a gap does not appear.

Przemysław Ohrysko (Institute of Mathematics of the Polish Academy of Sciences)

Non-separability of the Gelfand space of measure algebras

In this talk I would like to present some recent results concerning the Banach algebra $M(G)$ of Borel regular measures on a locally compact Abelian group with the convolution product. Since it is well-known that the spectrum of a measure can be much bigger then the closure of the values of its Fourier– Stieltjes transform (the Wiener–Pitt phenomenon) it is natural to ask what kind of topological properties of the Gelfand space $\mathfrak{M}(M(G))$ are responsible for this unusual spectral behaviour. It follows immediately from the existence of the Wiener–Pitt phenomenon that the set Γ (the dual group) identified with Fourier–Stieltjes coefficients is not dense in $\mathfrak{M}(M(G))$. However, it is not clear if any other countable dense subset of this space exists. During my talk, I will disprove this fact – i.e. I will show the non-separability of the Gelfand space of measure algebras on any non-discrete locally compact Abelian group. This is the main result of the paper 'Non-separability of the Gelfand space of measure algebras' written in collaboration with Michał Wojciechowski and Colin C. Graham which will be published soon in Arkiv för Matematik and is available on arxiv.org with identifier: 1603.05864.

Jani Onninen (Syracuse University / University of Jyvaskyla)

Monotone Sobolev Mappings of planar domains

(joint work with Tadeusz Iwaniec)

An approximation theorem of Youngs (1948) asserts that a continuous map between compact oriented topological 2-manifolds (surfaces) is monotone if and only if it is a uniform limit of homeomorphisms. In this talk we discuss analogous approximation results for Sobolev mappings. These results are at the very heart of Geometric Function Theory (GFT) and Nonlinear Elasticity (NE). In both theories the mappings in question arise naturally as weak limits of energy-minimizing sequences of homeomorphisms. As a result of this, the energy-minimal mappings turn out to be monotone. We establish the existence of energy-minimal deformations within the class of Sobolev monotone mappings.

Adam Osękowski (University of Warsaw)

Inequalities for Beurling–Ahlfors operator and related Fourier multipliers

We will show how Bellman function method, combined with probabilistic representation of certain class of Fourier multipliers, yields various tight estimates for Beurling–Ahlfors operator as well as its real- and imaginary part. We will present several important examples, including strong, weak-type and logarithmic inequalities.

Nikolaos Pattakos (Karlsruhe Institute of Technology)

On existence of global solutions of the 1*-dimensional cubic NLS for initial data in the modulation space* $M_{p,q}$

We prove global wellposedness for the 1-dimensional cubic NLS in the modulation space $M_{p,p'}$ for p sufficiently close to 2. The idea is to split the initial data between two suitable function spaces and solve in each of them a different NLS and then combine the solutions. The method of splitting goes back to Bourgain.

Carlos Pérez (University of the Basque Country / Ikerbasque)

Extensions of the weak type $(1,1)$ *property of classical operators: some conjectures by E. Sawyer*

Muckenhoupt–Wheeden [2] in the seventies and Sawyer [5] in the eighties, established one-dimensional highly nontrivial extensions of the weak type $(1,1)$ property of the maximal function involving weights. These results were conjectured to hold for the Hilbert transform and for the maximal function in higher extensions. In the first part of this lecture we will survey about these conjectures that were proved and extended in different directions in [1] and [3]. In the second part we will discuss a recent joint work with Israel P. Rivera-Ríos [4] showing the intimate connection of these estimates with the commutators of singular integral operators with BMO functions.

[1] D. Cruz-Uribe, SFO, J.M. Martell and C. Pérez, *Weighted weak-type inequalities and a conjecture of Sawyer*, Int. Math. Research Notices, **30** (2005), 1849-1871.

[2] B. Muckenhoupt and R. Wheeden, *Some weighted weak-type inequalities for the Hardy-Littlewood maximal function and the Hilbert transform*, Indiana Math. J. **26** (1977), 801-816.

[3] S. Ombrosi and C. Pérez, *Mixed weak type estimates: Examples and counterexamples related to a problem of E. Sawyer.* To appear, Colloquium Mathematicum.

[4] C. Pérez and Israel P. Rivera-Ríos, *Three observations on commutators of Singular Integral operators with BMO functions,* to appear AWM-Springer Series, Proceedings in honor of Cora Sadosky.

[5] E. Sawyer,*A weighted weak type inequality for the maximal function*, Proc. Amer. Math. Soc., **93** (1985), 610-614.

Sandra Pott (Lund University)

Matrix weights: On the way to the linear bound

(joint work with Andrei Stoica)

In recent years, the attempts to prove sharp bounds for Calderon-Zygmund operators on weighted L^p spaces in terms of the A_p-characteristic of the weight has been an important driving force in Harmonic Analysis. After the work of many authors, this culminated with the proof of the conjectured linear bound for $p = 2$ for all Calderon–Zygmund operators by Tuomas Hytönen in 2010.

Matrix A_p weights were introduced by Treil and Volberg in 1996. Recently, the question of the validity of the linear bound for all Calderon–Zygmund operators in the matrix-weighted setting has attracted some interest. In the talk, I want to present the reduction of this question to the case of Haar multipliers and dyadic paraproducts. I also want to talk about the remaining obstacles, some of which have recently been resolved, and focus on the matrix techniques being used.

Alexander Pushnitski (King's College London)

An inverse spectral problem for self-adjoint positive Hankel operators

The famous theorem due to Megretskii, Peller and Treil describes all possible spectral types for self-adjoint bounded Hankel operators. However, the question of how to describe all Hankel operators with the given spectral type remains open. I will describe some progress in this direction, which is the result of recent joint work with Patrick Gerard (Orsay). We consider a special class of bounded self-adjoint Hankel operators, satisfying a certain positivity condition. Within this class, we give a complete description of all Hankel operators with the given spectral type.

Jan Rozendaal (Institute of Mathematics of the Polish Academy of Sciences)

Operator-valued (L^p, L^q)-Fourier multipliers

Although much of the theory of Fourier multipliers has focused on the (L^p, L^p)-boundedness of Fourier multiplier operators, for many applications it suffices that a Fourier multiplier is bounded from L^p to L^q with p and q not necessarily equal. Moreover, one can derive (L^p, L^q)-boundedness results for $p \neq q$ under different, and often weaker, assumptions than in the case $p = q$. For example, it turns out that one can obtain (L^p, L^q)-boundedness of Fourier multipliers without smoothness assumptions on the symbol of the multiplier, and without the UMD assumption on the underlying Banach spaces.

In this talk I will explain some recent results on the (L^p, L^q)-boundedness of operator-valued Fourier multipliers, and I will indicate some applications of these results to the stability theory for operator semigroups, functional calculus theory and Schur multipliers.

Ameur Seddik (University of Batna 2)

Moore–Penrose inverse and operator inequalities

Let $\mathfrak{B}(H)$ be the C*-algebra of all bounded linear operators acting on a complex Hilbert space H, and let $\mathcal{R}(H)$ denote the set of all operators in $\mathfrak{B}(H)$ with closed ranges. We shall show that the class of all selfadjoint operators in $\mathfrak{B}(H)$ with closed ranges multiplied by scalars is characterized by each of two following properties

$$\forall X \in \mathfrak{B}(H),\ \left\|SXS^{+} + S^{+}XS\right\| \geq 2\left\|SXS\right\|,\ (S \in \mathcal{R}(H))$$
$$\forall X \in \mathfrak{B}(H),\ \left\|S^2X + XS^2\right\| \geq 2\left\|SS^{+}XS^{+}S\right\|,\ (S \in \mathcal{R}(H))$$

and the class of all normal operators in $\mathfrak{B}(H)$ with closed ranges is characterized by each of the two following properties

$$\forall X \in \mathfrak{B}(H),\ \left\|SXS^{+}\right\| + \left\|S^{+}XS\right\| \geq 2\left\|SS^{+}XS^{+}S\right\|,\ (S \in \mathcal{R}(H))$$
$$\forall X \in \mathfrak{B}(H),\ \left\|S^2X\right\| + \left\|XS^2\right\| \geq 2\left\|SXS\right\|,\ (S \in \mathcal{R}(H))$$

As application of these characterizations in the case of $\dim H < \infty$ (here $\mathfrak{B}(H) = \mathcal{R}(H)$), the class of all selfadjoint operators in $\mathfrak{B}(H)$ multiplied by scalars is characterized by each of two following properties

$$\forall X \in \mathfrak{B}(H),\ \left\|SXS^{+} + S^{+}XS\right\| \geq 2\left\|SXS\right\|,\ (S \in \mathfrak{B}(H))$$
$$\forall X \in \mathfrak{B}(H),\ \left\|S^2X + XS^2\right\| \geq 2\left\|SS^{+}XS^{+}S\right\|,\ (S \in \mathfrak{B}(H))$$

and the class of all normal operators in $\mathfrak{B}(H)$ is characterized by each of the two following properties

$$\forall X \in \mathfrak{B}(H),\ \left\|SXS^+\right\| + \left\|S^+XS\right\| \geq 2\left\|SS^+XS^+S\right\|,\ (S \in \mathfrak{B}(H))$$

$$\forall X \in \mathfrak{B}(H),\ \left\|S^2X\right\| + \left\|XS^2\right\| \geq 2\left\|SXS\right\|,\ (S \in \mathfrak{B}(H))$$

Kristian Seip (Norwegian University of Science and Technology)

From Gál's theorem to extreme values of the Riemann zeta function

This talk is a mathematical journey that takes us from a prize problem of Erdős that was solved by Gál in 1949, to the following theorem of Bondarenko and myself: The maximum of $|\zeta(1/2+it)|$ on the interval $T^{1/2} \leq t \leq T$ is at least

$$\exp\left((1/\sqrt{2}+o(1))\sqrt{\log T \log\log\log T/\log\log T}\right)$$

when $T \to \infty$.

Mitsuhiro Shishikura (Kyoto University)

Quasiconformal deformation and cross-ratios

For a quasiconformal mapping on the plane, a sufficient condition for the map to be differentiable or conformal at a specified point was given by Gutlyanskii–Martio. We give a simple proof using the deformation of cross-ratio of four points of the sphere. We also discuss a related question on the quasiconfomal deformation of rational maps.

Michał Strzelecki (University of Warsaw)

The L^p-norms of the Beurling–Ahlfors transform on radial functions

I will explain how one can find the L^p-norm ($1 < p < \infty$) of the Beurling-Ahlfors transform restricted to the class of radial functions. The value of this norm was previously known for $1 < p \leq 2$ (Bañuelos and Janakiraman; Bañuelos and Osękowski; Volberg). The approach presented in the talk will be connected to a maximal martingale inequality and will be based on the Bellman function technique.

Christoph Thiele (Universität Bonn)

Quantitative norm convergence for two commuting transformations

Xavier Tolsa (ICREA / Universitat Autònoma de Barcelona)

The Riesz transform, quantitative rectifiabilty, and a two-phase problem for harmonic measure

In the first part of my talk I will explain a recent theorem by Girela–Sarrión and myself which relates the quantitative rectifiability of general Radon measures in the Euclidean space to the L^2 boundedness of the Riesz transform of codimension 1. This result is obtained by techniques analogous to the ones of a previous result by Nazarov, Volberg and myself in connection with the David–Semmes problem.

In the second part I will explain an application (by Azzam, Mourgoglou and myself) of the preceding result to solve a two-phase problem for harmonic measure posed by Chris Bishop in 1990. This asserts that, for disjoint domains in the Euclidean space whose boundaries satisfy a non-degeneracy condition, mutual absolute continuity of their harmonic measures implies absolute continuity with respect to surface measure and rectifiability in the intersection of their boundaries. Up to now, by a result of Kenig and Toro, it was only known that the preceding condition implies that the harmonic measure is concentrated in a set of Hausdorff codimension 1.

Bartosz Trojan (University of Wrocław)

Maximal estimates for discrete operators of Radon-types

Vasily Vasyunin (St. Petersburg Department of V. A. Steklov Institute of Mathematics of the Russian Academy of Sciences)

Recent results in Bellman function technique

I present no solution of any specific problem, no concrete Bellman function, the aim is to present a method of finding solution of a rather general class of extremal problems. This class includes such classical problems as the John–Nirenberg inequality for BMO-functions, the reverse Hölder inequality for Muckenhoupt weights, and many others.

The mentioned class describes the case when the Bellman equation is reduced to the homogeneous Monge–Ampère equation for two variables and the domain of the definition of the Bellman function is the difference of two concave planar domains. The process of finding the Bellman function of an extremal problem (i.e. solving the boundary value problem of the Monge–Ampère equation) is reduced to constructing a so-called foliation of the domain. We classify all possible local foliations and describe the way how to construct the global foliation from these local elements. The main parameter determining the global architecture of the foliation is the sign of the curvature of the boundary curve. We describe the evolution of the foliation when the domain is increasing. A paper (153 pages) devoted to a special case of this construction for the case of BMO space is accepted for publishing in "Memoirs of the American Mathematical Society" (for a preprint version see arXiv:1510.01010). A paper describing the general case is now in preparation.

All this work was made in collaboration with my young colleagues Paata Ivanisvili, Dmitriy Stolarov, Pavel Zatitskiy, and partially Nikolai Osipov.

Alexander Volberg (Michigan State University)

A_2-conjecture in non-homogeneous setting

Michał Wojciechowski (Institute of Mathematics of the Polish Academy of Sciences)

Isomorphism of spaces of BV-functions on simply connected planar domains

Błażej Wróbel (University of Wrocław)

A characterization of spherical multipliers on homogeneous trees

(joint work with Dario Celotto and Stefano Meda)

We characterise, for each p in $(1,\infty)\setminus\{2\}$, the class of L^p spherical multipliers on homogeneous trees in terms of L^p Fourier multipliers on the torus. This extends previous work of M. Cowling, S. Meda and A. G. Setti for homogeneous trees, and is related with the work of A. D. Ionescu on noncompact symmetric spaces.

Peter Yuditskii (Johannes Kepler University Linz)

Sum rules and Killip–Simon problem

One of the first (and therefore most important) theorems in the perturbation theory claims that for an arbitrary self-adjoint operator A there exists a perturbation B of the Hilbert-Schmidt class, which destroys completely the absolutely continuous (a.c.) spectrum of the initial operator A (von Neumann). However, if A is the discrete free 1-D Schrödinger operator and B is an arbitrary Jacobi matrix (of Hilbert-Schmidt class) the a.c. spectrum remains perfectly the same, that is, the interval $[-2, 2]$. Moreover, Killip and Simon described explicitly the spectral properties for such $A + B$ [Annals, 2003]. The proof was based on a certain sum rule, a kind of Parseval's identity for a non-linear Fourier transform. Jointly with Fedor Nazarov and Sasha Volberg we found sum rules in a very general form [IMRN, 2005]. Let us point out that this was absolutely restricted to perturbations of an operator with constant coefficients. Therefore, the next result of Damanik, Killip and Simon on perturbations with a periodic background was a real breakthrough in the subject [Annals, 2010]. Recall, that the spectrum of a periodic Jacobi matrix is a system of intervals of a very specific nature. Now, we found a way to work with an arbitrary system of intervals (i.e., with an almost periodic background). To this end we define and study a new integrable system, which we call Jacobi flow on GMP matrices.

Anna Zdunik (University of Warsaw)

Real analyticity for random dynamics of transcendental functions

(joint work with Volker Mayer and Mariusz Urbański)

Analyticity results of expected pressure and invariant densities in the context of random dynamics of transcendental functions are established. These are obtained by a refinement of work by H. H. Rugh, leading to a simple approach to analyticity.

Our main application states real analyticity for the variation of the dimension of so-called radial Julia set for suitable hyperbolic random systems of entire or meromorphic functions.

Scott Zimmerman (University of Pittsburgh)

Sobolev extensions of Lipschitz mappings into metric spaces

Wenger and Young proved that $(\mathbb{R}^m, \mathbb{H}^n)$ has the Lipschitz extension property for $m \leq n$ where $\mathbb{H}^n$ is the sub-Riemannian Heisenberg group. That is, for some $C > 0$, any L-

Lipschitz map from a subset of $\mathbb{R}^m$ into $\mathbb{H}^n$ can be extended to a CL-Lipschitz mapping on $\mathbb{R}^m$. In this talk, we will discuss Sobolev extensions of such Lipschitz mappings with no restriction on the dimension m. We will show that any Lipschitz mapping from a closed subset of $\mathbb{R}^m$ into $\mathbb{H}^n$ can be extended to a Sobolev mapping on a bounded domain containing the set. This result is then generalized to include mappings into any Lipschitz $(n-1)$-connected metric space.