Topological quantum groups and Hopf algebras

Titles and abstracts

14–18 November 2016

Yuki Arano, University of Tokyo
Representation theory of Drinfeld doubles
17 November, 15:30-16:30

As has been observed by many authors, the Drinfeld double of the q-deformation of a compact Lie group can be regarded as a quantization of the complexification of the original Lie group. Using this point of view, I will discuss irreducible unitary representations of these Drinfeld doubles and compare with the classical case. As an application, we prove the dual of a q-deformation of compact Lie group of rank equal or more than 2 has central property (T).

Marat Aukhadiev, University of Munster
Quantum inverse semigroups
16 November, 09:30-10:30

In this talk I will introduce a quantum analogue of inverse semigroups.

Classically, an inverse semigroup is a semigroup consisting of partial isometries. This is a generalization of the notion of group. The theory has attracted a lot of attention for the last decades. We will review examples coming from groups and geometry, connections with groupoid C*-algebras, and recent important results on Toeplitz C*-algebras of cancellative semigroups. I will give a quantum analogue of the notion and show what examples we have so far. Connections with quantum groupoids (weak C*-Hopf algebras) will be also discussed.

On the joint work with A. Buss and T. Timmermann.

Julien Bichon, Université Blaise Pascal
Quantum subgroups of low-dimensional quantum groups
15 November, 15:30-16:30

A classical theme in the theory of compact groups is the problem of of classifying the subgroups of the low-dimensional ones, with the emblematic $SU(2)$ and $SO(3)$ cases, leading, even nowadays, to spectacular mathematical developments, such as the McKay correspondence.

Podleś initiated the study of the quantum version of this problem in the nineties, and classified the quantum subgroups of quantum $SU(2)$. I will survey the results on the description of the quantum subgroups of several low-dimensional quantum groups (including S_4^+, $U_{-1}(2)$, $SU_{-1}(3)$, O_n^*) obtained in the last years with several collaborators (in alphabetic order: T. Banica, M. Dubois-Violette, S. Natale, S. Neshveyev, M. Yamashita, R. Yuncken).

Alexandru Chirvasitu, University of Washington
Dynamics on quantum groups
15 November, 14:00-15:00

We study the topological structure of the automorphism groups of compact quantum groups, showing among other things that the connected component of the automorphism group and the "inner" automorphism group coincide (a quantum Iwasawa theorem). We also prove that for compact matrix quantum groups the inner automorphism group is a compact Lie group and the outer automorphism group is discrete.

(joint with Issan Patri, Chennai Mathematical Institute)

Jonathan Crespo, Vrije Universiteit Brussel
Monoidal equivalence of locally compact quantum groups and applications
17 November, 14:00-15:00

In this talk, I will present some results from a recent preprint in collaboration with S. Baaj. We generalize to the case of regular locally compact quantum groups two important results concerning the actions of compact quantum groups (see the work of De Rijdt and Vander Vennet and that of Voigt). Let G_1 and G_2 be two regular locally compact quantum groups (in the sense of Kustermans and Vaes) monoidally equivalent (in the sense of De Commer). We develop an induction procedure and we build an equivalence of the categories, whose objects are the continuous actions of G_1 and G_2 on C*-algebras. As an application of this result, we obtain a canonical equivalence of the categories of equivariant KK-theory for actions of G_1 and G_2. We introduce and investigate a notion of actions on C*-algebras of measured quantum groupoids (in the sense of Enock and Lesieur) on a finite basis. The proof of the latter equivalence relies on a version of the Takesaki-Takai duality theorem for continuous actions of measured quantum groupoids on a finite basis.

Ludwik Dabrowski, Scuola Internazionale Superiore di Studi Avanzati
Noncommutative Borsuk-Ulam-type conjectures revisited
14 November, 14:00-15:30

Let H be the C*-algebra of a non-trivial compact quantum group acting freely on a unital C*-algebra A. Baum, Dabrowski and Hajac conjectured that there does not exist an equivariant *-homomorphism from A to the equivariant noncommutative join C*-algebra $A * H$. When A is the C*-algebra of functions on a sphere, and H is the C*-algebra of functions on $Z/2Z$ acting antipodally on the sphere, then the conjecture becomes the celebrated Borsuk-Ulam theorem. Recently, Chirvasitu and Passer proved the conjecture when H is commutative. In particular, in the setting of point-set topology, this allows us to deduce consequences for the Ageev conjecture concerning actions of p-adic integers on the Menger compacta. The main goal of this talk is to show how to extend the Chirvasitu-Passer result to a far more general setting assuming only that H admits a character different from the counit. We show how our result implies a noncommutative Brouwer fixed-point theorem and, in particular, the non-contractibility of such compact quantum groups. Finally, assuming that our compact quantum group is a q-deformation of a compact connected semisimple Lie group, we prove that there

exists a finite-dimensional representation of the compact quantum group such that, for any C*-algebra A admitting a character, the finitely generated projective module associated with $A * H$ via this representation is not stably free. (Based on joint work with P. M. Hajac and S. Neshveyev.)

Biswarup Das, University of Oulu
Admissibility of finite dimensional unitary corepresentations of a locally compact quantum group
17 November, 11:00-12:00

In this talk we will give an affirmative answer to the following open problem in the theory of locally compact quantum groups, which was first posed by P. Sołtan in 2010, and subsequently came up again in some of the recent works in harmonic analysis on quantum groups: *All finite dimensional unitary corepresentations of a locally compact quantum group are admissible i.e. they "factor" through corepresentations of matrix quantum groups.*

The talk will be split into two parts:

In the first part, we will present a gentle introduction to the theory of locally compact quantum group and associated objects like multiplicative unitary and corepresentations, as developed in the recent years, avoiding technicalities and mostly building upon the motivation coming from classical locally compact group. This will get us to the precise statement of the above conjecture.

The second part will be devoted to solving the problem, and will consist of technical bits. Upon discussing the main analytical hurdles and involved technicalities, we will construct certain analytical tools towards a possible solution. The path to solving this open problem would be through proving the following: "Fourier-Stieltjes algebra of a locally compact quantum group is always (strong Voiculescu) amenable" (a "quantum generalization" of the corresponding result for locally compact groups).

Amaury Freslon, Université Paris-Sud XI
Free quantum groups and torsion
16 November, 14:00-15:00

R. meyer and R. Nest introduced a few years ago a notion of torsion for quantum groups which is suitable for the study of their K-theory through Baum-Connes type ideas. Recently, Y. Arano and K. De Commer developped cominatorial tools which enable one to study this torsion in detail as soon as one has a nice description of the fusion ring of the quantum group. It is therefore natural to investigate the problem for the class of free quantum groups, which are built from noncrossing partitions and hence have a well-understood representation theory. I will in particular explain how one can completely describe the torsion of the free wreath product of an arbitrary compact quantum group by a quantum permutation group.

Debashish Goswami, Indian Statistical Institute
Quantum symmetry of classical spaces
18 November, 09:30-10:30

I'll discuss the following conjecture: there cannot be any faithful action of a genuine compact quantum group on a compact connected (classical) smooth manifold such that the action is

smooth in a natural sense. We'll explain the main steps of proof of this conjecture in an important special case when the action is isometric (i.e. commutes with the Hodge Laplacian) for some Riemannian structure on the manifold. Moreover, the conjecture is verified (without the isometry condition) for arbitrary finite dimensional compact quantum groups. We'll also mention some related results in the algebraic framework due to Etingof and Walton as well as in the metric space set-up by Chirvasitu.

Mehrdad Kalantar, University of Houston
Open quantum subgroups and induced representations
16 November, 11:00-12:00

We introduce the notion of open quantum subgroups of locally compact quantum groups, and then show the two proposed constructions of induced representations, by Kustermans and by Vaes, are both equivalent in the setting of open quantum subgroups to Rieffel's (rather simple) construction of induced representations in the context of C^*-algebras.
This talk is based on joint work with Paweł Kasprzak, Adam Skalski, and Piotr Sołtan.

Yulia Kuznetsova, Laboratoire de Mathématiques de Besançon
A duality construction for locally compact quantum groups not involving the Haar weight
17 November, 09:30-10:30

We construct a certain duality functor on a wide class of operator bialgebras including locally compact quantum groups. The dual of a LCQG A in this sense is the universal dual $\hat{A}_u$ of Kustermans; but applied to $\hat{A}_u$, the construction yields the universal algebra A_u of the original quantum group, and this is the main new point. In the classical case, it puts in duality the algebras $C_0(G)$ and $C^*(G)$ of a locally compact group G. The motivation is, from one side, conceptual: to show that there is no need to know the Haar measure/weight to construct the dual of a locally compact quantum group, thus contributing to the problem of defining a quantum group in a measure-independent way. From the other side, there is more practical outcome: from given von Neumann bialgebras, even rather badly behaved, the proposed functor yields bialgebras with relatively good properties.

Marco Matassa, University of Oslo
Differential two-forms from projections on quantum flag manifolds
18 November, 15:30-16:30

One popular approach to differential forms on quantum spaces is based on Hochschild homology. This is motivated by the identification of differential forms on a classical space M with the Hochschild homology of the algebra of functions on M. Although this definition is applicable to non-commutative algebras, explicit computations reveal that the situation is very degenerate in general. It has been shown by several authors that the situation is much better when appropriate twistings are introduced. However not many explicit results are available and not much is known about the situation in intermediate degrees. In this talk I will outline a general method to obtain explicit twisted 2-cycles from projections on quantum flag manifolds. Their non-triviality in homology will be shown by pairing them with explicit twisted 2-cocycles. As an example I will discuss the case of quantum Grassmannians.

Ehud Meir, University of Hamburg
Applications of geometric invariant theory to Hopf algebras
15 November, 11:00-12:00

In this talk I will describe how classification problems in the theory of Hopf algebras can be approached by tools from Geometric Invariant Theory. The main focus will be on finite dimensional semi-simple Hopf algebras, and Hopf-Galois extensions of the ground field with an arbitrary finite dimensional Hopf algebras (which can be thought of as the non-commutative geometry version of a principal bundles over a point). As a result of a theorem of Radford, it follows that geometric invariant theory can be applied to study these problems, and that a complete set of scalar invariants can be found. I will describe these invariants, and I will explain their connection to the Frobenius-Schur indicators, the Reshetikhin-Turaev invariants of three manifolds, and the Universal Coefficients Theorem.

Chiara Pagani, University of Gottingen
Noncommutative principal bundles through twist deformation
18 November, 14:00-15:00

In the algebraic setting of noncommutative geometry, principal bundles are described via Hopf-Galois algebra extensions. The groups of symmetries of the bundle (structure group, groups of bundle automorphisms,...) are given by Hopf algebras. In this seminar I will talk about a joint work with P. Aschieri, P. Bieliavsky and A. Schenkel, in which we study deformation quantization of principal bundles via Drinfeld twists. A twist on the Hopf algebra of the structure group generates a bundle with twisted fibers. A twist on a group of bundle automorphisms generates a bundle with twisted base space. Examples will also be presented.

Piotr M. Sołtan, University of Warsaw
Compact quantum group actions on discrete quantum spaces and quantum Clifford theory
16 November, 15:30-16:30

In joint work with Kenny De Commer, Paweł Kasprzak and Adam Skalski we investigate compact quantum group actions on discrete quantum spaces. In particular we find that such an action defines a natural equivalence relation on a classical set associated to a discrete quantum space. This equivalence relation turns up in the study of quantum group generalization of Clifford theory.

Wenqing Tao, Huazhong University of Science and Technology
Noncommutative differentials on quantum groups and pre-Lie algebras
18 November, 16:45-17:45

We study noncommutative differential theory in the new directions of generalised differentials, codifferentials and pre-Lie algebras. We show that left covariant differentials on U(g) are classified by Lie algebra 1-cocycles while group 1-cocycles correspond to codifferential structures on algebraic groups. Among various constructions, we show that first order calculus on a Hopf algebra extends canonically to all orders via the braided super-shuffle algebra. Finally, we will

show that the quantisation of a connected simply-connected Poisson-Lie group admits a left-covariant noncommutative differential structure at lowest deformation order if and only if the dual of its Lie algebra admits a pre-Lie algebra structure. This talk is based on the joint work with Shahn Majid.

Alfons Van Daele, KU Leuven
From Hopf algebras to topological quantum groups: choices, difficulties and annoynaces
14 November, 10:30-12:00

Quantum groups have been studied within several areas of mathematics and mathematical physics. This has led to different approaches. Not only different techniques are used, but there are also various different conventions.

Starting with Hopf algebras (where there is a general consensus about the concept), moving in the direction of topological quantum groups (where there is no such consensus), it is easy to get lost. Not only many difficulties have to be overcome, but also several choices must be made. The way this has been done is often confusing and causes some annoyances.

As an introductory lecture at this conference on '*Topological quantum groups and Hopf algebras*', I would like to discuss various aspects of these *choices, difficulties and annoyances* encountered on the road from Hopf algebras to topological quantum groups.

I will start with finite quantum groups, then discuss discrete and compact quantum groups, and finish with a discussion on different approaches to locally compact quantum groups.

Elmar Wagner, Universidad Michoacana de San Nicolás de Hidalgo
Noncommutative topology of the quantum complex plane
18 November, 11:00-12:00

One of the most frequent relations in quantum group theory is $xy = qyx$, where q denotes a (real) deformation parameter. Defining a *-structure by $x^* := y$, one obtains the relation of the quantum complex plane $xx^* = qx^*x$. Operator theorist call Hilbert space operators satisfying this equation *q-normal operators*. It is known that q-normal operators a never bounded. To study such quantum spaces in the C*-algebraic setting, one may apply Woronowicz's theory of C*-algebras generated by unbounded elements. In this talk we give a complete topological description of C*-algebras generated by q-normal operators including K-theory, K-homology and index pairings. It turns out that among all these C*-algebras only one has the same K-theoretical behavior as the classical counterpart. This happens if and only $\mathrm{spec}(|z|) = [0, \infty)$. (Joint work with Ismael Cohen.)

Shuzhou Wang, University of Georgia
Kirchberg's factorization property for discrete quantum groups
15 November, 16:45-17:45

In the 1960s, Takesaki discovered that the full group C*-algebra of the free group on two generators is non-nuclear using its left and right regular representations. Wassermann subsequently showed that the product of the left and right regular representation is continuous under the minimal/spatial tensor product. Using these ideas, Kirchberg defined the notion of discrete groups with factorization property and gave several equivalent characterizations of

this property, extending similar properties Connes developed in his paper on the classification of injective factors. So far little is known about discrete groups with factorization property beyond the work of Kirchberg.

In this talk, we define the notion of Kirchberg's factorization property for unimodular discrete quantum groups and indicate how results of Brannan-Collins-Vergnioux and Brown-Dykema can be used to show that the discrete duals of the universal orthogonal and unitary quantum groups have this property when the dimension of the fundamental representation of the latter is different from 3. (Joint work with Angshuman Bhattacharya.)

Milen Yakimov, Louisiana State University
Belavin-Drinfeld quantum groups
15 November, 09:30-10:30

All quasitriangular Poisson structures on simple Lie groups are given by the Belavin–Drinfeld classification from the early 80s. In 2000 Etingof, Schedler and Schiffmann constructed explicit formal quantizations of them. Using their result, we construct algebraic quantizations $\mathcal{O}_{T,\chi}(G)$ of the coordinate rings of the corresponding connected, simply connected algebraic groups G. They are Hopf algebras that deform each of the Belavin–Drinfeld Poisson structures on G. In this talk we will describe structure results for these Belavin–Drinfeld quantum groups, culminating with a Levi type decomposition for them. This is a joint work with Tim Hodges, University of Cincinnati.

Makoto Yamashita, Ochanomizu University
Drinfeld center, tube algebra, and representation theory for monoidal categories
16 November, 16:45-17:45

Motivated by recent progress on the approximation properties of quantum groups, we study harmonic analytic aspects of monoidal categories based on the notion of Drinfeld center of ind-objects for rigid C*- tensor categories, from the tube algebra and related constructs from 2-categorical formulation. Besides the obvious subfactor theoretic input from Ocneaunu's work, this is also motivated by Schauenburg's work on the weak Morita equivalence between tensor categorios, which suggests a general method to obtain permanence of harmonic analytic properties under such equivalence. Based on joint works with S. Neshveyev.

Robert Yuncken, Université Blaise Pascal
The reduced dual of a complex semisimple quantum group
17 November, 16:45-17:45

This talk will be an overview of the unitary representation theory of complex semisimple quantum groups, such as $\mathrm{SL}_q(n, \mathbf{C})$. These quantum groups are defined as the doubles of the compact semisimple quantum groups—the terminology reflects the fact that the double behaves in many ways like the complexification of a classical compact semisimple Lie group. We focus on the reduced dual, where we can describe the Plancherel measure and the structure of the reduced C^*-algebra. (Joint work with Christian Voigt.)