DISCRETE GROUP ACTIONS ON C*-ALGEBRAS AND RHO CLASSES OF UNITARY REPRESENTATIONS

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Let Γ be the fundamental group of a closed manifold M and $\alpha \colon \Gamma \to U_n$ a finite dimensional unitary representation, i.e. a flat unitary vector bundle over M. To these data, Atiyah, Patodi and Singer associated a class $[\alpha]$ in the group $K^1(M, \mathbb{R}/\mathbb{Z})$ and investigated it in relation to rho invariants of Dirac operators.

In this talk, we take an operator algebraic point of view on the rho class $[\alpha]$ with the goal of generalising it to a noncommutative setting.

We start showing that $[\alpha]$ admits a canonical construction, using von Neumann algebras, and that a crucial role is played by Atiyah's L^2 -index theorem for coverings. We rephrase this, using KK-theory, by saying that the Γ -algebra $C_0(\widetilde{M})$ is K-theoretically free and proper.

Then we look at a C^* -algebra A endowed with the action of a discrete group Γ : if A is K-theoretically free and proper, given a unitary representation $\alpha \colon \Gamma \to U_n$, we construct a canonical rho class in KK-theory with \mathbb{R}/\mathbb{Z} -coefficients. We exhibit natural classes of algebras satisfying this property. Based on joint work with Paolo Antonini and Georges Skandalis.