Noncommutative index theory

Titles and abstracts

24–28 October 2016

Francesca Arici, Radboud University, Nijmegen
Cuntz-Pimsner algebras, mapping cones and weighted lens spaces
25 October, 15:30-16:30

Cuntz-Pimsner algebras are universal C*-algebras associated to a C* correspondence and they encode dynamical information. In the case of a self-Morita equivalence bimodule they can be thought of as total spaces of a noncommtuative circle bundle. In the first part of the talk I will describe their relation with the mapping cone extension and the corresponding exact sequence. The second part of the talk will focus on a Cuntz-Pimsner model for quantum weighted lens spaces that allows one to compute their K-theory and K-homology groups.

Sara Azzali, University of Potsdam
Discrete group actions on C*-algebras and rho classes of unitary representations
27 October, 14:00-15:00

Let Γ be the fundamental group of a closed manifold M and $\alpha\colon \Gamma \to U_n$ a finite dimensional unitary representation, i.e. a flat unitary vector bundle over M. To these data, Atiyah, Patodi and Singer associated a class $[\alpha]$ in the group $K^1(M, \mathbb{R}/\mathbb{Z})$ and investigated it in relation to rho invariants of Dirac operators.

In this talk, we take an operator algebraic point of view on the rho class $[\alpha]$ with the goal of generalising it to a noncommutative setting.

We start showing that $[\alpha]$ admits a canonical construction, using von Neumann algebras, and that a crucial role is played by Atiyah's L^2-index theorem for coverings. We rephrase this, using KK-theory, by saying that the Γ-algebra $C_0(\widetilde{M})$ is K-theoretically free and proper.

Then we look at a C^*-algebra A endowed with the action of a discrete group Γ: if A is K-theoretically free and proper, given a unitary representation $\alpha\colon \Gamma \to U_n$, we construct a canonical rho class in KK-theory with $\mathbb{R}/\mathbb{Z}$-coefficients. We exhibit natural classes of algebras satisfying this property. Based on joint work with Paolo Antonini and Georges Skandalis.

Paul F. Baum, Penn State, IMPAN
K-homology and index theory on contact manifolds
24 October, 10:30-12:00

The Baum-Douglas isomorphism of topological and analytic K-homology can be taken as providing a framework within which the Atiyah-Singer index theorem can be extended to certain non-elliptic differential operators. This talk considers a class of non-elliptic differential operators that arise on compact contact manifolds. In particular, we will focus on solving the Heisenberg-elliptic index problem. This is joint work with Erik van Erp.

Paolo Bertozzini, Thammasat University
Spectral Geometries for Some Diffeologies
28 October, 11:00-12:00

Diffeologies, introduced by JM Souriau, are a vast generalization of smooth manifolds that, from the point of view of category theory, is much better behaved. Not much is known on the possible ways to describe such spaces in the language of spectral geometry and it is the purpose of this quite preliminary talk to explore some of the problems involved in such study. We will also see how categories of spectral triples might be of help in these matters and more. This is a joint work with F.Germ and R.Conti.

Tomasz Brzeziński, Swansea University
Topology and K-theory of noncommutative weighted projective spaces
25 October, 14:00-15:00

In this talk we present results obtained jointly with Simon Fairfax and Wojciech Szymanski. Extending earlier results of Hong and Szymanski we interpret all quantum lens spaces as graph C*-algebras and thus design an efficient method for calculating K-groups of quantum lens spaces. Some special cases (not studied earlier) serve as an illustration. Using the freeness of the actions of the circle group on quantum lens spaces that define quantum weighted projective spaces we compute the K-groups of the latter.

Branimir Ćaćić, University of New Brunswick
Spectral triples for discrete groups
28 October, 14:00-15:00

The Pontrjagin dual of an Abelian discrete group is a compact topological group, but if the Abelian discrete group is finitely generated, then its dual is actually a Lie group. The reduced group C*-algebra of a non-Abelian discrete group is certainly a compact quantum group, but under what conditions can we endow it with the structure of a noncommutative manifold? In this talk, I'll recall Connes's construction of spectral triples for discrete groups endowed with a proper length function, and then I'll discuss how to generalise this construction to discrete groups endowed with a proper array, e.g., groups with the Haagerup property together with a witnessing proper 1-cocycle, at the price of working with unbounded KK-cycles. This is joint work in progress with Steve Avsec.

Robin Deeley,University of Hawaii
Dynamically relevant inductive limits for certain Smale space C*-algebras
28 October, 09:30-10:30

Smale spaces are a class of hyperbolic dynamical system. The stable C*-algebra of a Smale space is obtained from an etale groupoid and when the original system is mixing it is simple, separable, nuclear, and stably finite. One would like to compute the K-theory of this C*-algebra. I will outline the construction of an explicit inductive limit decomposition of the stable C*-algebra when the stable sets are totally disconnected. From this, one can often compute the

K-theory. The talk will be example based. In particular, no knowledge of Smale spaces is required. This talk is based on joint work with Allan Yashinski.

Magnus Goffeng, University of Gothenburg
Unbounded KK-theory and KK-bordisms
26 October, 15:30-16:30

This talk treats the notion of bordism in unbounded KK-theory, due to Hilsum. We will in this talk see how bordism defines an equivalence relation on the unbounded KK-cycles and the equivalence classes form an abelian group. For a closed manifold M and a C*-algebra A, the bordism group of $(Lip(M), A)$-cycles contain the KK-group $KK_*(C(M), A)$ as a direct summand. Joint work with Robin Deeley and Bram Mesland.

Alexander Gorokhovsky, University of Colorado, Boulder
A Hilbert bundle description of differential K-theory
27 October, 09:30-10:30

We give a description of differential K-theory in terms of infinite dimensional Hilbert bundles. As an application we propose a construction of twisted differential K-theory. This is a joint work with J. Lott.

Bruno Iochum, Aix-Marseille University
Heat trace for Laplace type operators with non-scalar symbols
26 October, 11:00-12:00

A new way to compute the heat-trace coefficients for second-order elliptic differential operators with non-scalar symbols will be given. Regardless of the length of calculation, we answer the question: is it possible to give explicit formulas for these coefficients? (joint work with Thierry Masson)

Jens Kaad, Radboud University, Nijmegen
Operator *-correspondences: Representations and pairings with unbounded KK-theory
26 October, 16:45-17:45

In this talk I will describe a very general class of hermitian bimodules called operator *-correspondences. This kind of bimodules typically arise as the domain of a metric connection acting on a C*-correspondence. Relying on the representation theory of completely bounded multilinear maps we shall then see how operator *-correspondences can be represented as bounded operators on a Hilbert space. As a further application and motivation for introducing operator *-correspondences we will describe how they (under an extra compactness assumption) admit an explicit pairing with a suitable abelian monoid of twisted unbounded Kasparov modules. The talk is partly based on joint work with David Blecher and Bram Mesland.

Matthias Lesch, Universitt Bonn
Zeta-determinants of Sturm-Liouville operators with quadratic potentials at infinity
26 October, 14:00-15:00

This is a report on a recent paper with Luiz Hartmann and Boris Vertman. We consider Sturm-Liouville operators on a half line, with potentials that are growing at most quadratically at infinity. Such operators arise naturally in the analysis of hyperbolic manifolds, or more generally manifolds with cusps. We establish existence and a formula for the associated zeta-determinant in terms of the Wronski-determinant of a fundamental system of solutions adapted to the boundary conditions. Despite being the natural objects in the context of hyperbolic geometry, spectral geometry of such operators has only recently been studied in the context of analytic torsion.

Terry A. Loring, University of New Mexico
Emergent topology of insulators
26 October, 09:30-10:30

Many insulators can be studied via finite matrix models. The joint Clifford spectrum, found using a non-commutative Dirac operator, is often an interesting hypersurface. Proving this uses one of ten KO or KU indices of finite-volume systems. Ongoing work with Schulz-Baldes compares these indices map to older indices for infinite volume systems. The mathematics here is closely recent to work on emergent geometry in string theory.

Bram Mesland, Leibniz Universitat Hannover
KK-theory and cohomology of arithmetic groups
24 October, 14:00-15:30

Cohomology of arithmetic groups and its structure as a Hecke module plays a prominent role in modern number theory. Classically, the cohomology of an arithmetic group Γ can be studied geometrically through its action on the associated global symmetric space X. In low dimensions, such actions produce non-compact hyperbolic manifolds as quotient spaces, as well as dynamically complicated actions on the boundary of X. In joint work with M. H. Sengun, we show that the cohomology of Γ, as a Hecke module, can be captured by the K-groups of certain noncommutative C*-algebras which encode the action of Γ both on X and on its boundary. The Hecke operators can be rigidly defined as explicit classes in KK-theory acting on relevant K-groups in a way compatible with Morita equivalence and boundary maps. This provides a uniform framework to study the K-homology of arithmetic groups.

Ryszard Nest, University of Copenhagen
On algebraic index theorem for discrete group actions
27 October, 11:00-12:00

Suppose that A is a deformation quantization algebra of a symplectic manifold M with an action of a discrete group G. The equivariant algebraic index theorem is a formula expressing the trace of an idempotent p in the crossed product A with G. The result can be expressed in terms of a pairing of the principal symbol of p with certain equivariant characteristic classes of

the underlying manifold M. The equivariant characteristic classes are viewed as classes in the periodic cyclic cohomology of the crossed product by using the inclusion of Borel equivariant cohomology due to Connes. We will sketch the ideas leading to the proof of the corresponding index theorem. This is the ongoing joint work with Alexander Gorokhovsky and Niek de Klejin.

Mira A. Peterka, University of Pennsylvania
Classification of Complex Vector Bundles over Noncommutative Complex Projective Space
25 October, 16:45-17:45

We classify and construct all topoligical vector bundles on C*-algebraic Θ-deformed complex projective space of arbitrary dimension n assuming a generic diophantine condition holds between the entries of the skew-symmetric $n+1$ by $n+1$ matrix Θ.

Raphaël Ponge, Seoul National University
Noncommutative Geometry, Conformal Geometry, and Cyclic Homology of Group Actions on Manifolds
27 October, 15:30-16:30

In this talk I shall explain how to apply tools of noncommutative geometry to obtain a local index formula in conformal geometry that are take into account of an arbitrary group of conformal diffeomorphisms. This uses the framework of twisted spectral triples of Connes-Moscovici. This leads us to a construction of a whole new family of conformal invariants. These invariants are expressed in terms of equivariant characteristic classes. Their computation was the main impetus for the explicit calculation of the cyclic homology of crossed-product algebras associated with group actions on manifolds. We shall also report on this cyclic homology computation, which enables us to recover and extend earlier results of Baum-Connes, Brylinski-Nistor, Connes, Crainic, and Wassermann, among others.

Hessel Bouke Posthuma, Universiteit Van Amsterdam
Lie algebroids and index theory
27 October, 16:45-17:45

Lie algebroids are generalizations of Lie algebras and tangent bundles. In the first part of this talk I will present the computation of the Hochschild and cyclic homology of their universal enveloping algebras in terms of Lie algebroid cohomology, generalizing earlier results by Kassel (Lie algebras) and Wodzicki (tangent bundles). In these computations the Lie-Poisson structure on the dual of the Lie algebroid plays an important role. In the second part of the talk I will explain how, by finding a symplectic framework for this Poisson structure, we can derive an index theorem generalizing the index theorem for differential operators of Bressler-Nest-Tsygan. This talk is based on joint work with Arie Blom.

Christian Voigt, University of Glasgow
The Plancherel Formula for complex quantum groups
25 October, 09:30-10:30

We explain the structure of the reduced group C*-algebras of complex semisimple quantum groups, and discuss a connection to the Baum-Connes assembly map for classical complex groups.

Hang Wang, University of Adelaide
Character Formula for Discrete Series
25 October, 11:00-12:00

Weyl character formula describes characters of irreducible representations of compact Lie groups. This formula can be obtained using geometric method, for example, from the Atiyah-Bott fixed point theorem or the Atiyah-Segal-Singer index theorem. Harish-Chandra character formula, the noncompact analogue of the Weyl character formula, can also be studied from the point of view of index theory. We apply orbital integrals on K-theory of Harish-Chandra Schwartz algebra of a semisimple Lie group G, and then use geometric method to deduce Harish-Chandra character formulas for discrete series representations of G. This is work in progress with Peter Hochs.