Scale invariant O'Hara energies and s-p harmonic maps

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We report on advances of geometric knot energies defined by Jun O'Hara:

The O'Hara energies [2, 3] acting on periodic Lipschitz maps $\gamma : \mathbb{R}/\mathbb{Z} \to \mathbb{R}^3$ are defined as

$$\mathcal{O}^{\alpha,p}(\gamma) = \int_{\mathbb{R}/\mathbb{Z}} \int_{\mathbb{R}/\mathbb{Z}} \left(\frac{1}{|\gamma(x) - \gamma(y)|^\alpha} - \frac{1}{d_\gamma(x,y)^\alpha} \right)^{\frac{p}{2}} |\gamma'(x)||\gamma'(y)| dx dy. \tag{1}$$

Here $d_\gamma(x,y)$ is the distance between $\gamma'(x)$ and $\gamma'(y)$ along the curve γ.

If $\alpha p = 4$ these energies are independent of the specific parametrization of the knot.

Up to now, only the case $(\alpha, p) = (2,2)$ was understood - in particular due to the seminal work [1]. Their geometric arguments crucially rely on the Möbius invariance of $\mathcal{O}^{\alpha,p}$ which only holds if $(\alpha, p) = (2,2)$.

We present a regularity result for critical knots and in particular minimizers for general $\alpha p = 4$. The main point is that the derivative γ' of any knot γ is a critical point of a new energy $\mathcal{E}$ if and only the knot γ is critical for $\mathcal{O}^{\alpha,p}$. Since by reparametrization we can always assume γ' to map into a sphere, we use methods developed for degenerate fractional harmonic maps into spheres [4] to obtain regularity.

Joint work with S. Blatt, Ph. Reiter.

References

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