

# Eigenvalues of Fourier multipliers and generalized tight frames

BARTŁOMIEJ SIUDEJA

University of Oregon, USA

We study spectral functionals of Laplace related Fourier multipliers, that is a family of operators with symbols  $f(|\xi|^2)$ . As  $f$ , we consider arbitrary Bernstein functions [5], leading to generators of subordinate Brownian motions. We also look at polynomials, leading bi-Laplacian (plate problems, [1]) and higher order Laplacians.

In all these cases we find geometrically sharp estimates for the sums of eigenvalues of affinely transformed symmetric domains. For example we show that balls are the maximizers of the sums of eigenvalues among ellipses, assuming appropriate “moment of mass” scaling. It is worth noting that Faber-Krahn (spectral isoperimetric) inequality states that balls are minimizer of the smallest eigenvalue assuming mass scaling. This work extends an earlier approach co-developed with Richard Laugesen [3].

We achieve our results by reducing variational problems to existence of overdetermined bases (tight frames) with strange powers of the distances in the Parseval identity, as in Bachoc-Ehler [2]. Informally, we look at sets of vectors (or matrices) for which a version of the Pythagorean theorem holds with powers other than 2. For example, start by projecting a planar vector onto directions given by cubic roots of unity. Sum the fourth powers of the lengths of the projections to get a number that is proportional (with a universal constant) to the fourth power of the length of the initial vector.

As a corollary of our results about sums of eigenvalues we also obtain bounds for traces of heat kernels and zeta functions. The talk is based on the paper [4].

## REFERENCES

- [1] **M. S. Ashbaugh, R. D. Benguria, and R. S. Laugesen**, *Inequalities for the first eigenvalues of the clamped plate and buckling problems*, General inequalities, 7 (Oberwolfach, 1995), Internat. Ser. Numer. Math., vol. 123, Birkhäuser, Basel, 1997, pp. 95–110. [MR1457272](#)
- [2] **C. Bachoc and M. Ehler**, *Tight  $p$ -fusion frames*, Appl. Comput. Harmon. Anal. **35** (2013), no. 1, 1–15. [MR3053743](#)
- [3] **R. Laugesen and B. Siudeja**, *Sums of Laplace eigenvalues: rotations and tight frames in higher dimensions*, J. Math. Phys. **52** (2011), no. 9, 093703, 13 pp. [MR2867829](#)
- [4] **B. Siudeja** *Generalized tight  $p$ -frames and spectral bounds for Laplace-like operators*, to appear in Appl. Comput. Harmon. Anal. [doi:10.1016/j.acha.2015.08.001](#)
- [5] **R. L. Schilling, R. Song, and Z. Vondraček**, *Bernstein functions*, second ed., de Gruyter Studies in Mathematics, vol. 37, Walter de Gruyter & Co., Berlin, 2012, Theory and applications. [MR2978140](#)