Bessel heat kernel estimates

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Consider the Bessel differential operator

$$L^{(\mu)} = \frac{d^2}{dx^2} + \frac{2\mu+1}{x}\frac{d}{dx}, \quad \mu \in \mathbf{R}$$

and the related heat kernels on interval $[0,1)$ and half-line $(1,\infty)$, i.e. the fundamental solutions of a partial differential equation

$$(\partial_t - L^{(\mu)})u = 0,$$

where we consider Dirichlet condition at 1 and Dirichlet or Neumann condition at 0. In the case of interval and $\mu > -1$ the problem relates to the classical Fourier-Bessel expansions and the Fourier-Bessel heat kernel. I will discuss probabilistic methods used to find the sharp two-sided estimates of the heat kernels for the whole range of space and time parameters obtained recently in [1], [2] and [3]. Unlike most of the results of this type, the exponential behavior of the kernels is here described explicitly i.e. there are no different constants in the exponential factors in the lower and upper bounds.

Joint works with Kamil Bogus and Grzegorz Serafin.

References

[1] K. Bogus, J. Małecki, *Sharp estimates of transition probability density for Bessel process in half-line.* Potential Anal. Volume 43, Issue 1, p. 1-22. (2015)

[2] K. Bogus, J. Małecki, *Heat kernel estimates for the Bessel differential operator in half-line.*. Math. Nachr. 1–11 (2016) DOI:10.1002/mana.201500163

[3] J. Małecki, G. Serafin, T. Zorawik, *Fourier-Bessel heat kernel estimates.* J. Math. Anal. Appl. Volume 439, Issue 1, p. 91-102 (2016)