Intrinsic contractivity properties of relativistic Schrödinger operators

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The classical hypercontractivity, supercontractivity and ultracontractivity properties of the ground state transformed (also called *intrinsic*) semigroups corresponding to the fairly general self-adjoint operators can be refined by the concept of the ground state domination. This observation applies to the semigroups of nonlocal Schrödinger operators based on generators of symmetric jump-paring Lévy processes with locally Kato-class potentials [1]. As a consequence, we obtain for a large class of potentials that, on the one hand, supercontractivity and ultracontractivity, on the other hand, hypercontractivity and asymptotic ultracontractivity of the transformed semigroup are equivalent properties. This is in stark contrast to classical Schrödinger operators, for which all these properties are known to be different.

In my talk, I will present the explicit joint characterization of hypercontractivity and asymptotic ultracontractivity of the ground state transformed semigroups corresponding to the fairly general class of nonlocal Schrödinger operators. In particular, I will discuss the examples of *fractional Schrödinger operators* $H = (-\Delta)^{\alpha/2} + V$ and *relativistic Schrödinger operators* $H = (-\Delta + m^{2/\alpha})^{\alpha/2} - m + V$, $\alpha \in (0,2)$, $m > 0$.

Joint work with M. Kwaśnicki and J. Lőrinczi.

References

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