Gradient perturbations of non-local operators

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Let L be the generator of a Lévy process $(X_t)_{t\geq 0}$ in $\mathbb{R}^d$, $d \in \mathbb{N}$, without Gaussian component, that is

$$Lf(x) = \langle \zeta, \nabla f(x) \rangle + \int_{\mathbb{R}^d} \big(f(x+z) - f(x) - \mathbf{1}_{|z|<1} \langle z, \nabla f(x) \rangle \big) \, \nu(dz) \, ,$$

where $\zeta \in \mathbb{R}$ and ν is a Lévy measure, i.e., $\nu(\{0\}) = 0$ and $\int_{\mathbb{R}^d} (1 \wedge |z|^2) \nu(dz) < \infty$. We consider ν that is comparable with an *isotropic unimodal* Lévy measure ν_0 such that the corresponding characteristic exponent $\psi_0(\xi) = \int_{\mathbb{R}^d} (1 - \cos \langle \xi \cdot z \rangle) \nu_0(dz)$ satisfies certain *weak scaling* conditions ([1]). We enhance perturbation methods [4] to construct and analyze a fundamental solution of the equation $\partial_t = L + b(t,x) \cdot \nabla_x$, i.e, a function $\tilde{p}$ such that

$$\int_s^\infty \int_{\mathbb{R}^d} \tilde{p}(s,x,u,z) \big[\partial_u + L + b(u,z) \cdot \nabla_z \big] \phi(u,z) \, dz du = -\phi(s,x) \, ,$$

holds for all $s \in \mathbb{R}$, $x \in \mathbb{R}^d$ and $\phi \in C_c^\infty(\mathbb{R} \times \mathbb{R}^d)$. The gradient (or the drift) term b is assumed to satisfy integrability condition that allows for b beyond an enlarged gradient Kato class. The study requires preliminary results such as an upper bound for the solution of $\partial_t = L$, that is the density $p_t(x)$ of the distribution of X_t, as well as for its gradient $|\nabla p_t(x)|$.

The talk is based on a joint work [2] with Tomasz Grzywny and covers previous results of [3].

References

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