Asymptotics of the solutions in nonlocal spatially continuous parabolic Anderson-type models with Poissonian interaction: annealed and quenched behaviour

Katarzyna Pietruska-Pałuba
University of Warsaw, Poland

We consider probabilistic solutions to the following spatially continuous parabolic Anderson-type problem

$$\frac{\partial u}{\partial t} + H^{\omega} u = 0, \quad u(0,x) \equiv 1,$$

where

$$H^{\omega} = -L + V^{\omega}$$

is a *random nonlocal Schrödinger operator* based on the generator L of a Lévy process $(X_t)_{t\geq 0}$ with random Poissonian potential V^{ω}.

The operator L is a pseudodifferential operator that can be defined analytically via its Fourier transform:

$$\widehat{Lf}(\xi) = -\psi(\xi)\widehat{f}(\xi), \quad f \in D(L) := \left\{ f \in L^2(\mathbb{R}^d) : \psi \widehat{f} \in L^2(\mathbb{R}^d) \right\}$$

with

$$\psi(\xi) = \xi \cdot A\xi + \int_{\mathbb{R}^d \setminus \{0\}} (1 - \cos(\xi \cdot z))\nu(dz),$$

where A is a symmetric non-negative definite $d \times d$ matrix and $\nu(dz)$ is an infinite symmetric Lévy measure on $\mathbb{R}^d \setminus \{0\}$, $d \geq 1$.

The solutions of this problems $u^{\omega}(t,x)$, can be defined probabilitstically using the Feynman-Kac formula. With this tool, we investigate their long-time behaviour. We consider two types of asymptotics: after averaging with respect to the random medium (the so-called annealed asymptotics), and also almost-sure with respect to the random medium (the so-called quenched asymptotics). For Brownian motion, such problems were addressed by Donsker and Varadhan (annealed), Sznitman (quenched). In that case, the annealed asymptotics and the quenched asymptotics are different.

For Levy processes, the two rates may differ, but they can coincide as well, which is a new phenomenon.

We will discuss precise rate functions, both annealed and quenched, for particular processes (e.g. stable, relativistic).

The results of this talk were obtained jointly with Kamil Kaleta (Wrocław).

References

[1] K. Kaleta, K. Pietruska-Pałuba, *The quenched asymptotics for nonlocal Schrödinger operators with Poissonian potentials*, preprint 2016 (available at arXiv:1601.05597)