Quasi-geostrophic equation in $\mathbb{R}^2$

MARIA KANIA-BŁASZCZYK
University of Silesia in Katowice, Poland

Solvability of Cauchy's problem in $\mathbb{R}^2$ for subcritical quasi-geostrophic equation is discussed here in phase spaces $H^s(\mathbb{R}^2)$ with $s > 1$. The problem has the form:

$$\theta_t + u \cdot \nabla\theta + \kappa(-\Delta)^\alpha\theta = f, \quad x \in \mathbb{R}^2,\, t > 0,$$

$$\theta(0, x) = \theta_0(x), \quad x \in \mathbb{R}^2,$$

where θ represents the potential temperature, $\kappa > 0$ is a diffusivity coefficient, $\alpha \in (\frac{1}{2}, 1]$ is a fractional exponent, and $u = (u_1, u_2)$ is the *velocity field* determined by θ through the relation: $u = (-R_2\theta, R_1\theta)$, where $R_i, i = 1, 2$ are the *Riesz transforms.* A solution to that equation in *critical case ($\alpha = \frac{1}{2}$)* is obtained next as a limit of the H^s-solutions to subcritical equations when the exponent α of $(-\Delta)^\alpha$ tends to $\frac{1}{2}^+$. We also discuss solvability of the critical problem with Dirichlet boundary condition in bounded domain $\Omega \subset \mathbb{R}^2$, when $\|\theta_0\|_{L^\infty(\Omega)}$ and $\|f\|_{L^\infty(\Omega)}$ are small.

Joint work with the Tomasz Dłotko and Chunyou Sun.

REFERENCES

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