The l^1-anisotropic total variation flow

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We consider the l^1-anisotropic total variation flow in the plane, corresponding formally to the equation

$$u_t = \operatorname{div}(\operatorname{sgn} u_{x_1}, \operatorname{sgn} u_{x_2}).$$

This is a member of the family of anisotropic total variation flows $u_t = \operatorname{div}(\partial\varphi(Du))$ with $\varphi = \|\cdot\|_1$, whose discretised versions have been particularly often applied to image denoising and decomposition as an alternative to the isotropic case ($\varphi = \|\cdot\|_2$).

We prove that the flow preserves the class of functions piecewise constant on rectangles, in fact we provide explicit description of evolution in this class. It is crucial to understand what exactly happens at time instances when several regions where the function is constant merge and non-local phenomenon of breaking may be exhibited, leading to expansion of the jump set, which is not observed in the isotropic case.

Nevertheless, approximating continuous functions with piecewise constant functions, we are able to show that continuous data stay continuous. In fact, a large class of moduli of continuity is preserved by the flow.

An essential ingredient is a lemma, where a class of Cheeger problems for l^1-anisotropic perimeter is solved.