On the equivalence between the Dirichlet and the Neumann problem for the Laplace operator

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We give a representation of the solution of the Neumann problem for the Laplace operator on the n-dimensional unit ball $\mathbb{U} \subset \mathbb{R}^n$ in terms of the solution of an associated Dirichlet problem. The result is the following.

Theorem 1. *Assume $\phi : \partial\mathbb{U} \to \mathbb{R}$ is continuous and satisfies $\int_{\partial\mathbb{U}} \phi(z)\, \sigma_0(dz) = 0$. If u is the solution of the Dirichlet problem for $\mathbb{U}$ with boundary condition $\varphi = \phi$ on $\partial\mathbb{U}$, then*

$$U(z) = \int_0^1 \frac{u(\rho z)}{\rho} d\rho, \qquad z \in \overline{\mathbb{U}}, \tag{1}$$

is the solution to the Neumann problem for $\mathbb{U}$ with $U(0) = 0$.

This representation is extended to other operators besides the Laplacian, to smooth simply connected planar domains, and to the infinite-dimensional Laplacian on the unit ball of an abstract Wiener space, providing in particular an explicit solution for the Neumann problem in this case.

As an application, we derive an explicit formula for the Dirichlet-to-Neumann operator, which may be of independent interest. Time depending, we will also present some recent results concerning the extension of this correspondence to generalized solutions of Dirichlet and Neumann problems.

The talk is based on [1], and on some other recent results.

Joint work with L. Beznea (Institute of Mathematics of the Romanian Academy, Bucharest, Romania) and N. R. Pascu (Kennesaw State University, Marietta, SUA).

References

[1] L. Beznea, M. N. Pascu, N. R. Pascu, *An Equivalence Between the Dirichlet and the Neumann Problem for the Laplace Operator*, Potential Analysis (to appear), online at Springer.