Long-time behavior in a doubly nonlocal Fisher-KPP equation

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We consider the following Fisher-KPP-type equation, with non-local both diffusion and nonlinear part

$$\begin{cases} \dfrac{\partial u}{\partial t}(x,t) = \varkappa^+(a^+ * u)(x,t) - \varkappa^- u(x,t)(a^- * u)(x,t) - mu(x,t), & x \in \mathbb{R}^d, t > 0, \\ u(x,0) = u_0(x), & x \in \mathbb{R}^d, \end{cases}$$

where a^+ and a^- are probability kernels. Under minimal conditions on the coefficients, we prove existence, uniqueness, and uniform space-time boundedness of the positive solution.

We investigate the existence and main properties of the front of propagation. Namely, we describe behavior of the profile of the solution, for large time. It is shown that in the case when a^+ is anisotropic and exponentially integrable (light-tailed) and u_0 is fast decaying, the front propagates with a constant speed. We study existence, uniqueness, and asymptotic behavior of monotone traveling waves for the equation in this case. If a^+ and u_0 are radially symmetric and at least one of them is slowly decaying (heavy-tailed), we give an explicit formula, which describes the acceleration of the front.

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