Heat kernels of regularly/slowly varying convolution semigroups

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We study the asymptotic properties and estimates of the convolution semigroups which are generated by isotropic unimodal Lévy processes. Let $\mathbf{X} = (X_t : t \geq 0)$ be an isotropic unimodal Lévy process on $\mathbb{R}^d$ with the characteristic exponent ψ. In the recent paper [1], the estimates of the heat kernels $p(t, x)$ were studied under the assumption that ψ has lower and upper Matuszewska indices strictly between 0 and 2. More detailed information can be obtained whenever stronger assumption about the behaviour of ψ is imposed. In the first part of my talk I present main results of [2]. For instance, we proved that if ψ varies regularly at infinity with index $\alpha \in (0, 2)$ then

$$\lim_{(x, t\psi(|x|^{-1})) \to 0} \frac{p(t, x)}{t|x|^{-d}\psi(|x|^{-1})} = \mathcal{A}_{d, \alpha}. \tag{1}$$

Moreover, the asymptotic (1) implies that ψ varies regularly at infinity with index $\alpha \in (0, 2)$.

The natural question arises about the asymptotic behaviour and estimates of the semigroup for the endpoints $\alpha \in \{0, 2\}$. In this talk we fill that gap in the theory for $\alpha = 0$. We provide a solution for a large class of slowly varying symbols including geometric stable and iterated geometric stable cases. Namely, we study processes with symbols belonging to de Haan class Π_ℓ^∞ associated to a function ℓ slowly varying at infinity which is equivalent to regular variation with index $-d$ at the origin of the density of Lévy's measure of the process. We derive several asymptotic results describing the behaviour of the heat kernel $p(t, x)$. Moreover, we present several estimates for the heat kernel under various assumptions for unimodal and isotropic processes, which improve the existing results even in the case of subordinate Brownian motion (see [3]).

Joint work with Michał Ryznar and Bartosz Trojan.

References

[1] Krzysztof Bogdan, Tomasz Grzywny and Michał Ryznar, *Density and tails of unimodal convolution semigroups*, J. Funct. Anal. **266** (2014), no. 6, 3543–3571.

[2] Wojciech Cygan, Tomasz Grzywny and Bartosz Trojan, *Asymptotic behavior of densities of unimodal convolution semigroups*. To appear in Trans. Amer. Math. Soc..

[3] Tomasz Grzywny, Michał Ryznar and Bartosz Trojan, *Asymptotic behaviour and estimates of slowly varying convolution semigroups*. Preprint 2016 (arxiv.org: 1606.04178).