# Estimates for solutions of fractal Burgers equation

Tomasz Jakubowski
Wrocław University of Science and Technology, Poland

Let $d \in \mathbb{N}$ and $\alpha \in (1,2)$. We consider the following fractal Burgers equation

$$\begin{cases} u_t - \Delta^{\alpha/2} u + b \cdot \nabla \left(u|u|^q\right) = 0, & t > 0,\ x \in \mathbb{R}^d, \\ u(0,x) = u_0(x), \end{cases} \tag{1}$$

where $b \in \mathbb{R}^d$ is a constant vector, $q = (\alpha - 1)/d$ and $u_0$ is a nonnegative function from $L^1(\mathbb{R}^d)$. We also consider the case $q > (\alpha-1)/d$ under the additional condition $u_0 \in L^\infty(\mathbb{R}^d)$. These type of equations were considered in [1], [2], [3], where the existence and some basic properties of the solution $u(t,x)$ of this equation were obtained. Recently, in [4], it was proved that for arbitrary constant $M > 0$ and $u_0(x) = M\delta_0(x)$, we have

$$C^{-1} p(t,x) \le u(t,x) \le C p(t,x),$$

where $p(t,x) = (2\pi)^{-d} \int_{\mathbb{R}^d} e^{-ix\cdot\xi} e^{-t|\xi|^\alpha} d\xi$ is the fundamental solution of the equation $u_t = \Delta^{\alpha/2} u$.

The aim of this talk is to present the following main result of the paper [5].

**Theorem 1.** *Let $d \ge 1$ and $\alpha \in (1,2)$. Let $u(t,x)$ be the solution of the equation (1) with $q \ge (\alpha-1)/d$. There exists a constant $C = C(d, \alpha, u_0, b)$ such that*

$$C^{-1} P_t u_0(x) \le u(t,x) \le C P_t u_0(x), \qquad t > 0,\ x \in \mathbb{R}^d,$$

*where*

$$P_t f(x) = \int_{\mathbb{R}^d} p(t, y-x) f(y) dy.$$

## References

[1] P. Biler, T. Funaki, and W. A. Woyczynski. Fractal Burgers equations. *J. Differential Equations*, 148(1):9–46, 1998.

[2] P. Biler, G. Karch, and W. A. Woyczyński. Critical nonlinearity exponent and self-similar asymptotics for Lévy conservation laws. *Ann. Inst. H. Poincaré Anal. Non Linéaire*, 18(5):613–637, 2001.

[3] P. Biler, G. Karch, and W. A. Woyczyński. Asymptotics for conservation laws involving Lévy diffusion generators. *Studia Math.*, 148(2):171–192, 2001.

[4] T. Jakubowski and G. Serafin. Stable estimates for source solution of critical fractal Burgers equation. *Nonlinear Anal.* 130, 396–407, 2016

[5] T. Jakubowski and G. Serafin. Pointwise estimates for solutions of fractal Burgers equation. Preprint, 2015