

On the construction of a Lévy-type process associated with a stable-dominated Lévy-type kernel

VICTORIA KNOPOVA

V.M. Glushkov Institute of Cybernetics of National Academy of Science of Ukraine

Consider the integro-differential equation operator L , defined on functions from the space $C_\infty^2(\mathbb{R}^d)$ of twice continuously differentiable functions with vanishing at infinity derivatives as

$$Lf(x) := \int_{\mathbb{R}^d} (f(x+u) - f(x) - u \cdot \nabla f(x) 1_{\{|u| \leq 1\}}) \nu(x, du), \quad (1)$$

where the kernel $\nu(x, du)$ is comparable with a Lévy measure ν_0 , such that

$$\nu_0(A) = \int_{\mathbb{S}^d} \int_0^\infty 1_A(r\theta) r^{-1-\alpha} dr \mu_0(d\theta), \quad A \subset \mathbb{R}^d, \quad \alpha \in (0, 2), \quad (2)$$

$\mu(A) = \mu(-A)$ for $A \subset \mathbb{R}^d$, and for small r

$$\nu_0(B(x, r)) \leq r^\gamma, \quad |x| = 1, \quad \gamma + \alpha > d.$$

Our goal is to show that the extension of $(L, C_\infty^2(\mathbb{R}^d))$ is the generator of a Feller semigroup $(T_t)_{t \geq 0}$, on the space $C_\infty(\mathbb{R}^d)$ of continuous functions vanishing at infinity, which in turn corresponds to a Feller process X_t :

$$T_t f(x) = \mathbb{E}^x f(X_t), \quad f \in C_\infty(\mathbb{R}^d), \quad t > 0.$$

Our construction relies on the parametrix method. As a part of the construction, we prove that the process X admits the transition probability density, derive the the upper bound on it, and prove some regularity properties.

The talk relies of the joint work with Krzysztof Bogdan and Pawel Sztonyk.

REFERENCES

- [1] K. Bogdan, V. Knopova, P. Sztonyk. *Transition densities of parastable Markov processes*. In preparation.