Approximation of subordinate semigroups via the Chernoff Theorem

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The talk is devoted to approximation of evolution semigroups generated by some Markov processes (and hence to approximation of transition probabilities of these processes) with the help of the Chernoff theorem. This theorem provides conditions for a family (just a family, not a semigroup!) of bounded linear operators $(F(t))_{t\geq 0}$ on a Banach space X to approximate the semigroup $(e^{tL})_{t\geq 0}$ with a given generator L in X via the formula $e^{tL} = \lim_{n\to\infty} [F(t/n)]^n$. This formula is called *Chernoff approximation of the semigroup* e^{tL} *by the family* $(F(t))_{t\geq 0}$. The most important condition of the Chernoff Theorem is the coincidence of the derivative of $F(t)$ at $t = 0$ with the generator L.

The considered in the talk semigroups correspond to processes obtained by subordination (i.e., by a time-change) of some original (parent) Markov processes with respect to some Lévy processes with a.s. increasing paths - subordinators (they play the role of the new time). If the semigroup, corresponding to a parent Markov process, is not known explicitly then neither the subordinate semigroup, nor even the generator of the subordinate semigroup are known explicitly too. In spite of the last fact, it is still possible to construct Chernoff approximations for such semigroups (in the case when subordinators have either known transitional probabilities, or known and bounded Lévy measure) under the condition that the parent semigroups are already Chernoff-approximated (see [1]). As it has been shown in the recent literature (see, e.g., [2]-[8] and references therein), this condition is fulfilled for several interesting classes of evolution semigroups. This fact allows to use the constructed Chernoff approximations for subordinate semigroups, in order to obtain explicit approximations for semigroups corresponding to subordination of Feller processes and (Feller type) diffusions in Euclidean spaces, star graphs and Riemannian manifolds. The obtained approximations can be used for direct calculations and simulation of stochastic processes. In several cases the obtained approximations are given as iterated integrals of elementary functions and hence provide representations of the considered semigroups by the so-called *Feynman formulae*. Representations of evolution semigroups by Feynman formulae usually lead to related Feynman-Kac representations. The method of Chernoff approximation can be also interpreted as construction of Markov chains approximating a given Markov process.

References

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